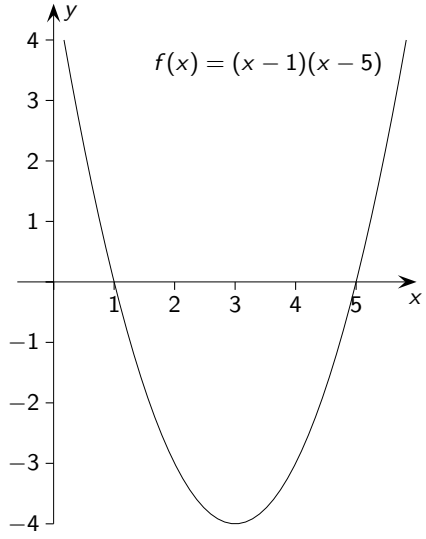
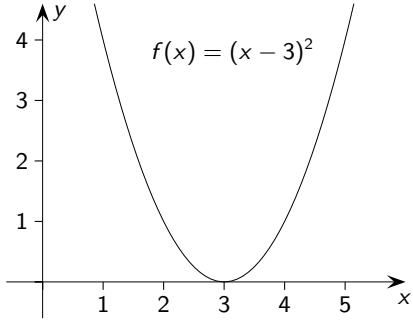
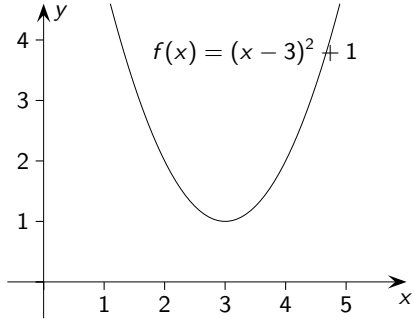
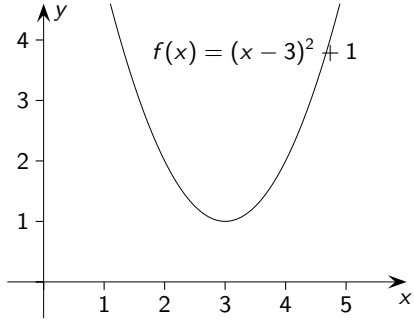
Zerlegung in Linearfaktoren

groolfs.de


y
x
f(x) = (x − 1)(x − 5)
4
3
2
1
−1
−2
−3
−4
1
2
3
4
5

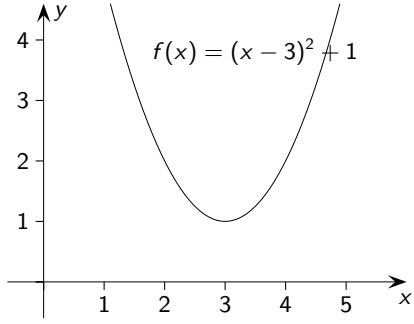

y
4
3
2
1
$f(x) = (x-3)^2$
1
2
3
4
5
x


y
4
3
2
1
$f(x) = (x-3)^2 + 1$
1
2
3
4
5
x



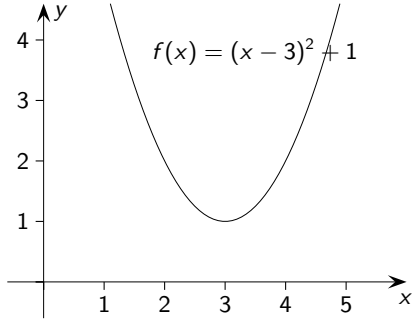
Die Zerlegung eines Polynoms 2. Grades richtet sich nach der Anzahl der Nullstellen (2, 1 oder 0) der zugehörigen Polynomfunktion (Parabel).

Polynom 2. Grades $ax^2 + bx + c$



Die Zerlegung eines Polynoms 2. Grades richtet sich nach der Anzahl der Nullstellen (2, 1 oder 0) der zugehörigen Polynomfunktion (Parabel).

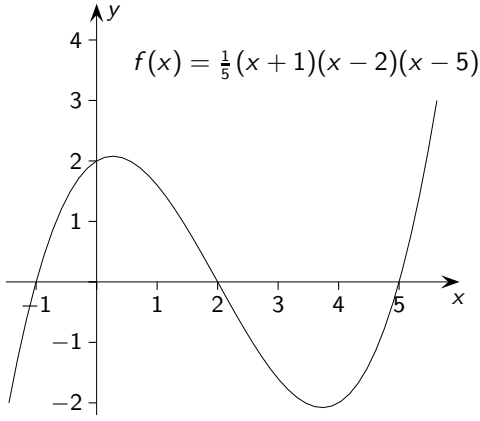
Für 2 Nullstellen gilt z. B.: $3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-1)(x-5)$

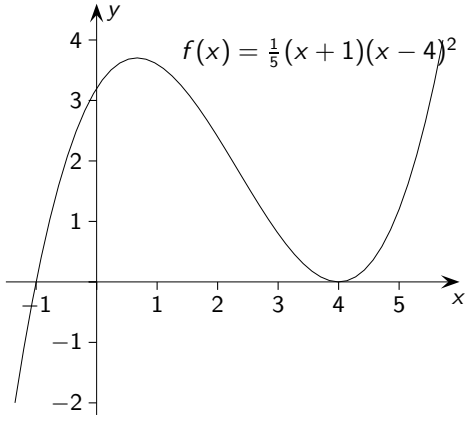


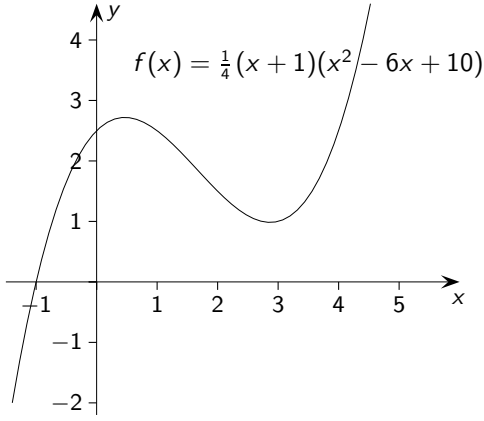
Die Zerlegung eines Polynoms 2. Grades richtet sich nach der Anzahl der Nullstellen (2, 1 oder 0) der zugehörigen Polynomfunktion (Parabel).

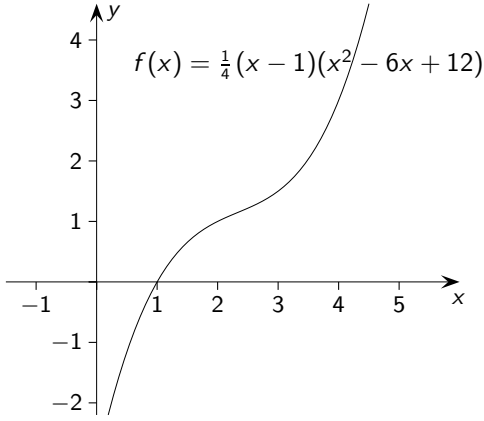
Für 2 Nullstellen gilt z. B.: $3x^2 - 18x + 15 = 3(x^2 - 6x + 5) = 3(x-1)(x-5)$

$(x-3)^2 + 1$ kann nicht in Linearfaktoren zerlegt werden.

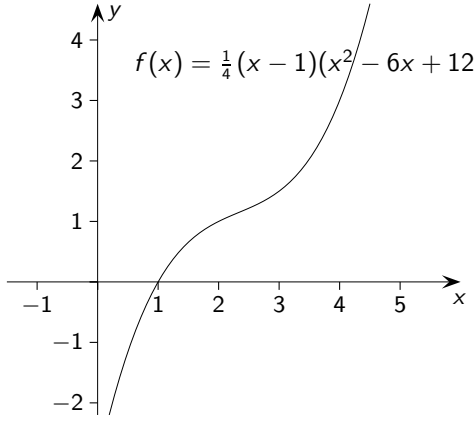

$f(x) = \frac{1}{5}(x+1)(x-2)(x-5)$
y
x
4
3
2
1
−1
−2
−1
1
2
3
4
5


$f(x) = \frac{1}{5}(x+1)(x-4)^2$
y
x
4
3
2
1
−1
−2
−1
1
2
3
4
5

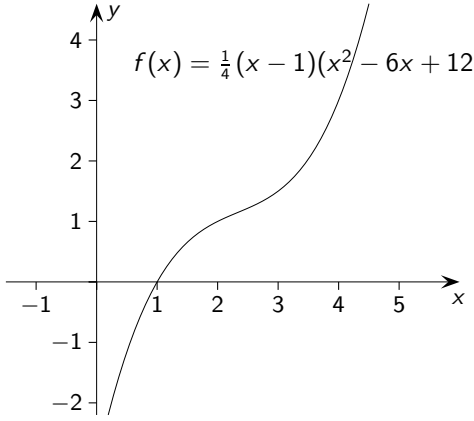

y
x
$f(x) = \frac{1}{4}(x+1)(x^2-6x+10)$
4
3
2
1
−1
−2
−1
1
2
3
4
5


y
x
$f(x) = \frac{1}{4}(x-1)(x^2-6x+12)$
4
3
2
1
−1
−2
−1
1
2
3
4
5

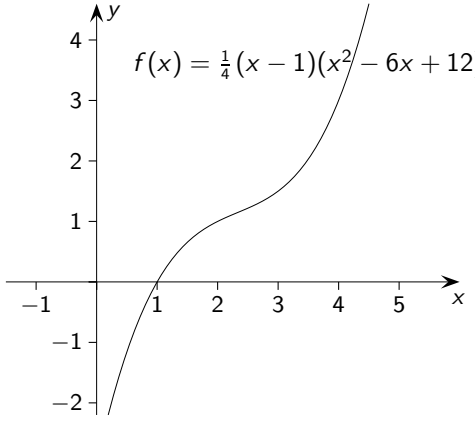
Polynom 3. Grades $ax^3 + bx^2 + cx + d$



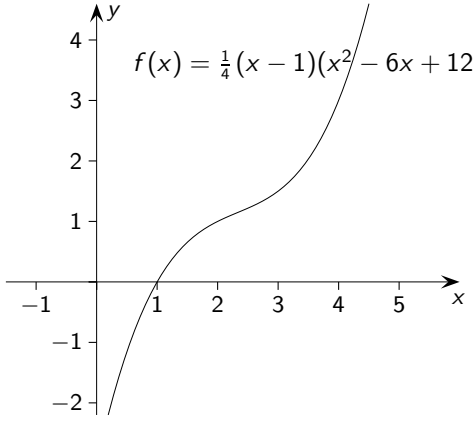
Die Zerlegung eines Polynoms 3. Grades richtet sich nach der Anzahl der Nullstellen



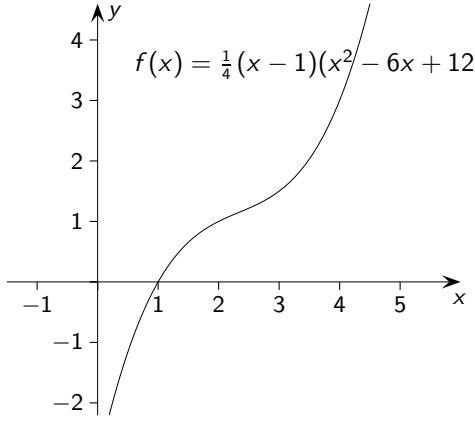
Die Zerlegung eines Polynoms 3. Grades richtet sich nach der Anzahl der Nullstellen (3, 2 oder 1) der zugehörigen Polynomfunktion. Mindestens eine Nullstelle ist vorhanden.



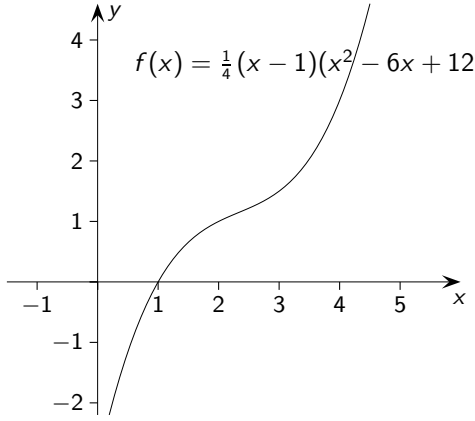
Für eine Zerlegung ist die 1. Nullstelle zu erraten. Für mögliche weitere Nullstellen ist eine



Für eine Zerlegung ist die 1. Nullstelle zu erraten. Für mögliche weitere Nullstellen ist eine Polynomdivision durchzuführen, z. B.:



$$\frac{x^3 - 7x^2 + 8x + 16}{x + 1} = \ldots = x^2 - 8x + 16 \quad \text{und} \quad x^2 - 8x + 16 = (x-4)^2 \quad \text{mit der } pq\text{-Formel}$$



$$\frac{x^3 - 7x^2 + 8x + 16}{x + 1} = \ldots = x^2 - 8x + 16 \quad \text{und} \quad x^2 - 8x + 16 = (x - 4)^2 \quad \text{mit der } pq\text{-Formel}$$

Insgesamt $x^3 - 7x^2 + 8x + 16 = (x + 1)(x - 4)^2$

$$(x^3 + x^2 - 10x + 8) : (x - 2) =$$

$$(x^3 + x^2 - 10x + 8) : (x-2) = x^2$$

$$(x^3 + x^2 - 10x + 8) : (x-2) = x^2$$
$$-(\qquad\qquad)$$

$$(x^3 + x^2 - 10x + 8) : (x-2) = x^2$$
$$-(x^3 \qquad)$$

$$(x^3 + x^2 - 10x + 8) : (x-2) = x^2$$
$$\underline{-\,(x^3 - 2x^2)}$$

$$
\begin{array}{l}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 \\
\underline{-(x^3 - 2x^2)} \\
\qquad 3x^2
\end{array}
$$

$$
\begin{array}{l}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 \\
\underline{-(x^3 - 2x^2)} \\
\qquad 3x^2 - 10x \\
\qquad\quad \underline{\qquad\qquad} \\
\qquad\qquad\quad \underline{\qquad\qquad}
\end{array}
$$

$$
\begin{array}{l}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x \\
\underline{-(x^3 - 2x^2)} \\
\qquad 3x^2 - 10x \\
\qquad\quad \underline{\qquad\qquad} \\
\qquad\qquad\quad \underline{\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x \\
\underline{-\,(x^3 - 2x^2)} \\
\qquad\quad 3x^2 - 10x \\
\quad \underline{-\,(\qquad\quad)} \\
\qquad\quad \\
\qquad\quad \underline{\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x \\
\underline{-(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-(3x^2 \quad\quad)} & \\
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x \\
\underline{-(x^3 - 2x^2)} \\
\qquad\quad 3x^2 - 10x \\
\quad \underline{-(3x^2 - \ 6x)} \\
\qquad\quad\ \ \underline{\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x & \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x & \\
\underline{\qquad\qquad} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x \\
\underline{-(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-(3x^2 - \ 6x)} & \\
-4x + 8 & \\
\underline{\qquad\qquad} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-(3x^2 - \ 6x)} & \\
-4x + 8 & \\
\underline{\qquad\qquad} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-(3x^2 - \ 6x)} & \\
-4x + 8 & \\
\underline{-(\qquad\qquad)} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-(3x^2 - \ 6x)} & \\
-4x + 8 & \\
\underline{-(-4x \quad)} & \\
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x + 8 & \\
\underline{-\,(-\,4x + 8)} & \\
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x + 8 & \\
\underline{-\,(-\,4x + 8)} & \\
0 &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x + 8 & \\
\underline{-\,(-\,4x + 8)} & \\
0 &
\end{array}
$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) =$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x + 8 & \\
\underline{-\,(-\,4x + 8)} & \\
0 &
\end{array}
$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 \\
\underline{-\,(x^3 - 2x^2)} & \\
3x^2 - 10x & \\
\underline{-\,(3x^2 - \ 6x)} & \\
-\,4x + 8 & \\
\underline{-\,(-\,4x + 8)} & \\
0 &
\end{array}
$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x$$

$$
\begin{array}{rl}
(x^3 + x^2 - 10x + 8) : (x-2) = x^2 + 3x - 4 & \\
\underline{-\,(x^3 - 2x^2)} & \\
\qquad 3x^2 - 10x & \\
\quad \underline{-\,(3x^2 - \ 6x)} & \\
\qquad\qquad -\,4x + 8 & \\
\qquad\quad \underline{-\,(-\,4x + 8)} & \\
\qquad\qquad\quad 0 &
\end{array}
$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) =$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2$$
$$-(\qquad\qquad)$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2$$
$$\underline{-(x^3 \qquad\quad)}$$

$$(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2$$
$$\underline{-(x^3 + 5x^2)}$$

$$
\begin{array}{l}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 \\
\underline{-(x^3 + 5x^2)} \\
\qquad x^2
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 \\
\underline{-(x^3 + 5x^2)} \\
\qquad x^2 + 3x \\
\qquad \underline{\qquad\qquad} \\
\qquad\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{l}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad x^2 + 3x \\
\qquad\quad \underline{\qquad\qquad} \\
\qquad\quad\ \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{l}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad\quad x^2 + 3x \\
\quad \underline{-(\qquad\quad)} \\
\\
\quad\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad\quad x^2 + 3x \\
\quad \underline{-(x^2 \qquad)} \\
\\
\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad\quad x^2 + 3x \\
\quad \underline{-(x^2 + 5x)} \\
\\
\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad\quad x^2 + 3x \\
\quad \underline{-(x^2 + 5x)} \\
\qquad\quad -2x \\
\\
\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x \\
\underline{-(x^3 + 5x^2)} \\
\qquad\quad x^2 + 3x \\
\quad \underline{-(x^2 + 5x)} \\
\qquad\quad -2x - 10 \\
\\
\quad \underline{\qquad\qquad\qquad}
\end{array}
$$

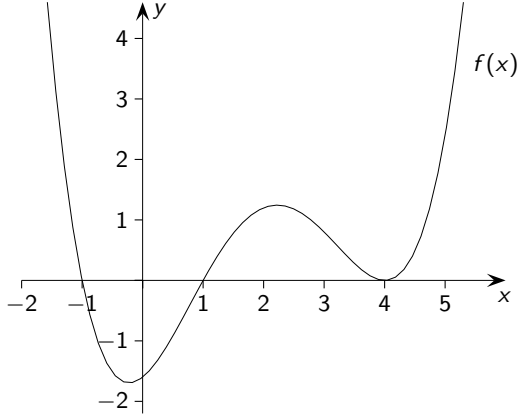
$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2 \\
\underline{-(x^3 + 5x^2)} & \\
x^2 + 3x & \\
\underline{-(x^2 + 5x)} & \\
-2x - 10 & \\
\underline{\qquad\qquad} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2 \\
\underline{-\,(x^3 + 5x^2)} & \\
\qquad\quad x^2 + 3x & \\
\quad \underline{-\,(x^2 + 5x)} & \\
\qquad\quad -2x - 10 & \\
\quad \underline{-\,(\qquad\qquad)} &
\end{array}
$$

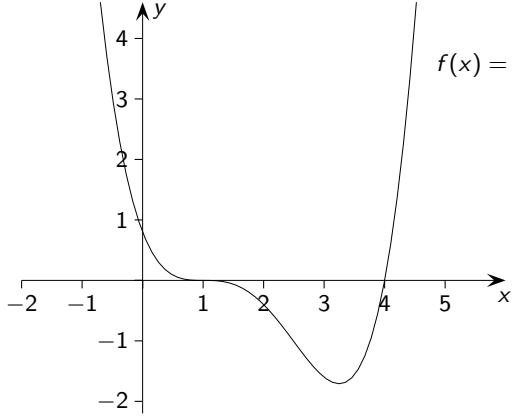
$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2 & \\
\underline{-(x^3 + 5x^2)} & \\
\qquad\quad x^2 + 3x & \\
\quad \underline{-(x^2 + 5x)} & \\
\qquad\quad -2x - 10 & \\
\quad \underline{-(-2x\qquad)} &
\end{array}
$$

$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2 \\
\underline{-\,(x^3 + 5x^2)} & \\
\qquad\quad x^2 + 3x & \\
\qquad \underline{-\,(x^2 + 5x)} & \\
\qquad\qquad\quad -\,2x - 10 & \\
\qquad\quad \underline{-\,(-\,2x - 10\,)} & \\
\end{array}
$$

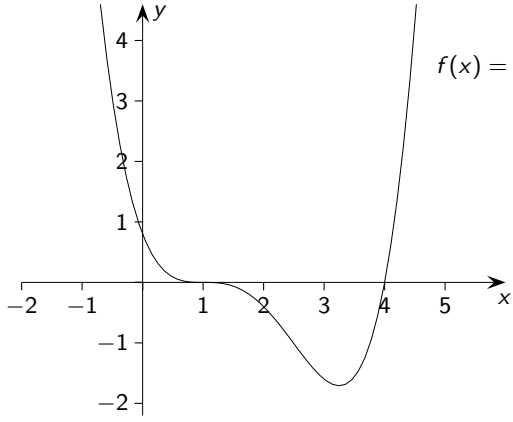
$$
\begin{array}{rl}
(x^3 + 6x^2 + 3x - 10) : (x+5) = x^2 + x - 2 & \\
\underline{-\,(x^3 + 5x^2)} & \\
\qquad\quad x^2 + 3x & \\
\qquad\underline{-\,(x^2 + 5x)} & \\
\qquad\qquad\quad -\,2x - 10 & \\
\qquad\quad\underline{-\,(-\,2x - 10\,)} & \\
\qquad\qquad\qquad\quad 0 &
\end{array}
$$



$$f(x) = \frac{1}{10}(x+1)(x-1)(x-4)^2$$

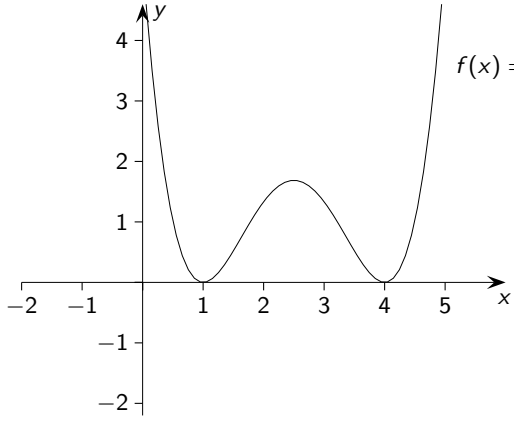


$$f(x) = \frac{1}{5}(x-1)^3(x-4)$$

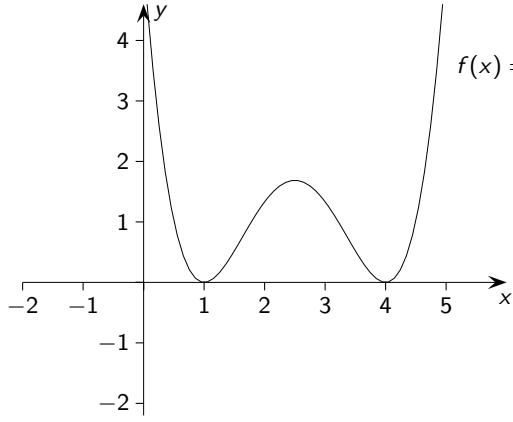


$f(x) = \frac{1}{5}(x-1)^3(x-4)$

In der Umgebung von $x = 1$ ähnelt der Graph dem verschobenen Graphen von $y = -x^3$.
Beachte: In dieser Umgebung ist der Faktor $(x - 4)$ negativ.



$$f(x) = \frac{1}{3}(x-1)^2(x-4)^2$$



In den Umgebungen von $x = 1$ und $x = 4$ ähnelt der Graph dem verschobenen Graphen von $y = x^2$. In diesen Umgebungen ist jeweils der übrige quadratische Term positiv.