

Kurvendiskussion $f(x) = -3x^3 + 12x^2 - 12x$

1. Nullstellen:

Bed.: $f(x) = 0$

$$-3x^3 + 12x^2 - 12x = 0 \quad | : (-3)$$

$$x^3 - 4x^2 + 4x = 0$$

$$x(x^2 - 4x + 4) = 0$$

$$x_1 = 0$$

$$x^2 - 4x + 4 = 0$$

$$x_{2/3} = 2 \pm \sqrt{4 - 4}$$

$$x_{2/3} = 2$$

2. Steigungen in den Nullstellen:

$$f'(x) = -9x^2 + 24x - 12$$

$$f'(0) = -12$$

$$f'(2) = 0$$

3. Extrema:

notwendige Bed.: $f'(x) = 0$

$$-9x^2 + 24x - 12 = 0 \quad | : (-9)$$

$$x^2 - \frac{8}{3}x + \frac{4}{3} = 0$$

$$x_{1/2} = \frac{4}{3} \pm \sqrt{\frac{16}{9} - \frac{12}{9}}$$

$$x_{1/2} = \frac{4}{3} \pm \frac{2}{3}$$

$$x_1 = 2$$

$$x_2 = \frac{2}{3}$$

$$\text{Max}(2 \mid 0)$$

$$\text{Min}\left(\frac{2}{3} \mid -\frac{32}{9}\right)$$

Minimum oder Maximum?

$$f''(x) = -18x + 24$$

$$f''(2) = -36 + 24 < 0$$

$$f''\left(\frac{2}{3}\right) = -12 + 24 > 0$$

4. Gleichung der Tangente im Punkt

$W\left(\frac{4}{3} \mid ?\right)$:

$$f\left(\frac{4}{3}\right) = -\frac{16}{9}$$

$$m = f'\left(\frac{4}{3}\right) = 4$$

$$y = 4x + b$$

$$-\frac{16}{9} = 4 \cdot \frac{4}{3} + b \implies b = -7\frac{1}{9}$$

$$y = 4x - 7\frac{1}{9}$$

$A(0 \mid 0)$:

$$y = -12x$$

