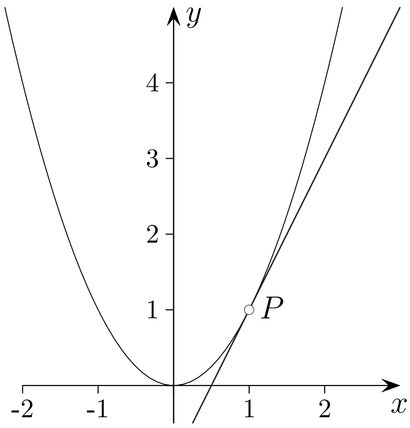
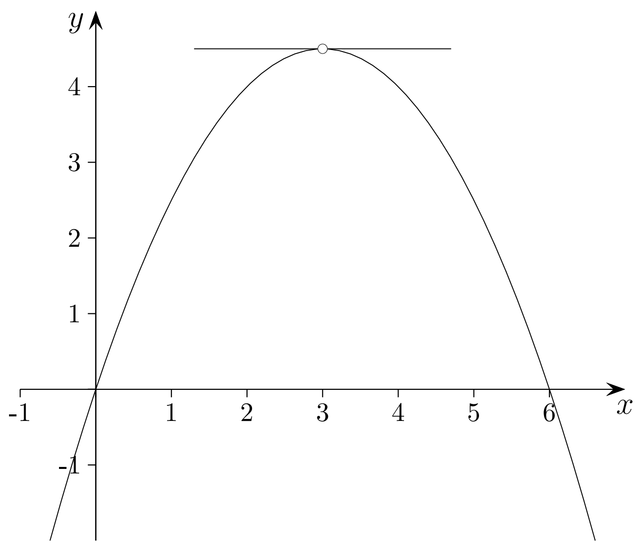
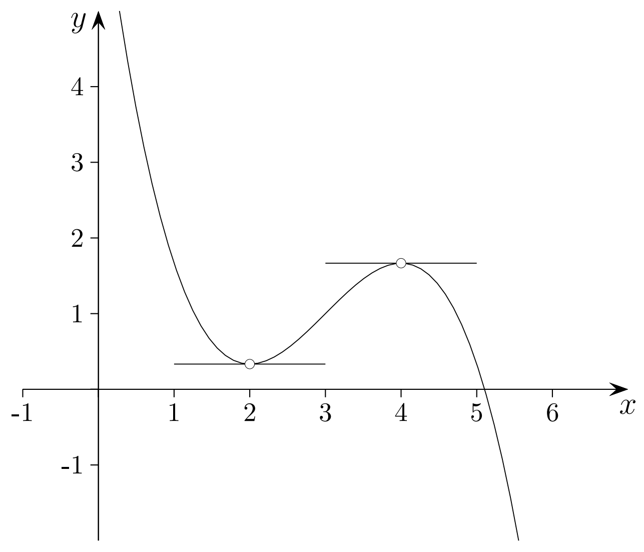
Tangentensteigung

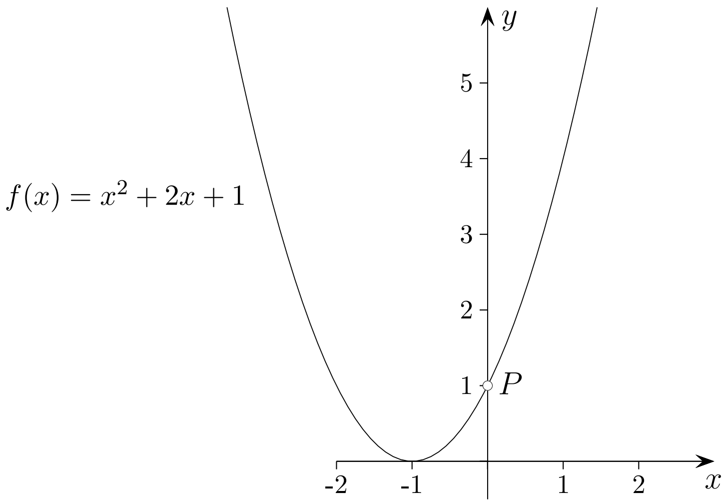


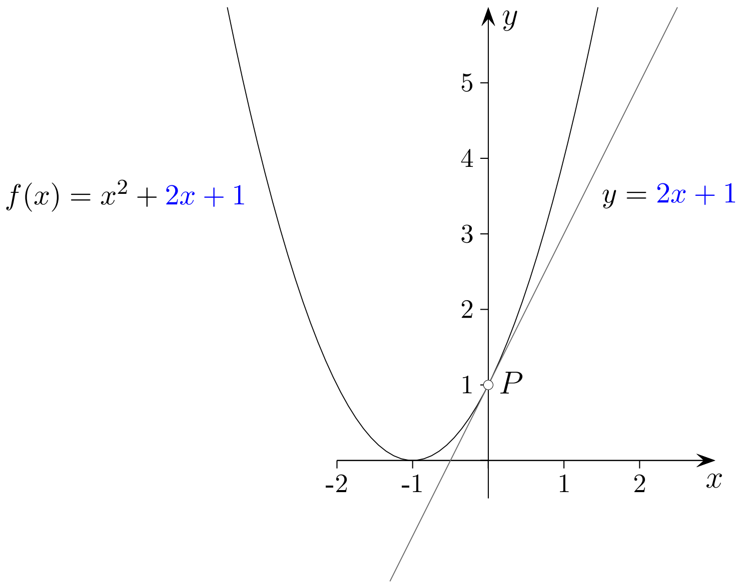


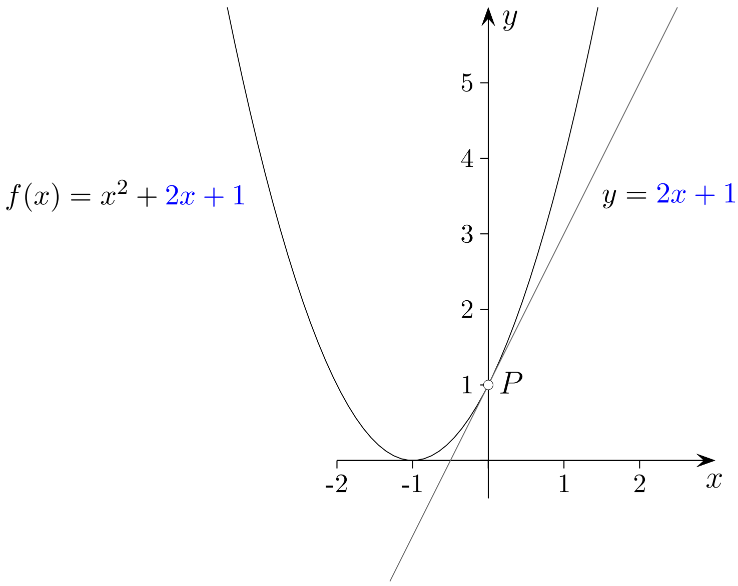
y
4
3
2
1
-1
1
2
3
4
5
6
x
-1



y
4
3
2
1
-1
1
2
3
4
5
6
x
-1

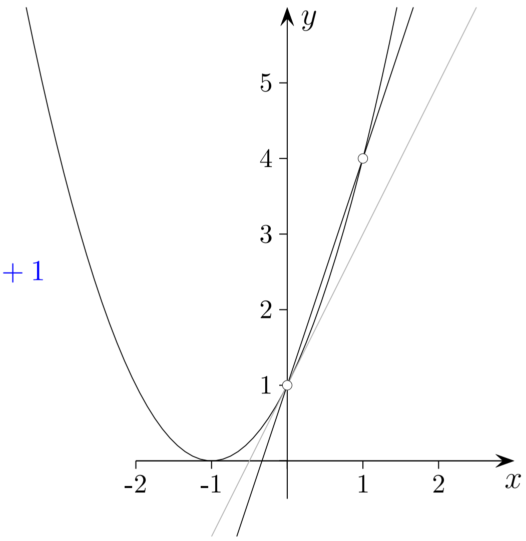

$f(x) = x^2 + 2x + 1$
y
5
4
3
2
1
P
-2
-1
1
2
x


y
5
4
3
2
1
P
-2
-1
1
2
x
$f(x) = x^2 + 2x + 1$
$y = 2x + 1$



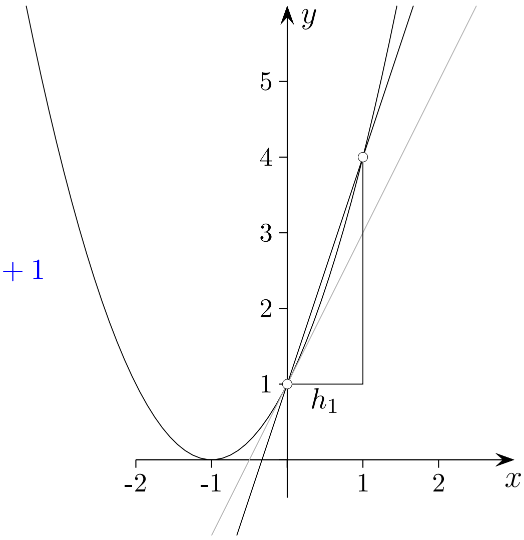
$$\begin{aligned} x^2 + 2x + 1 &= 2x + 1 \\ x^2 &= 0 \\ x &= 0 \end{aligned}$$

$f(x) = x^2 + 2x + 1$



Punkte auf dem Graphen
$(0|1)$ und $(1|4)$

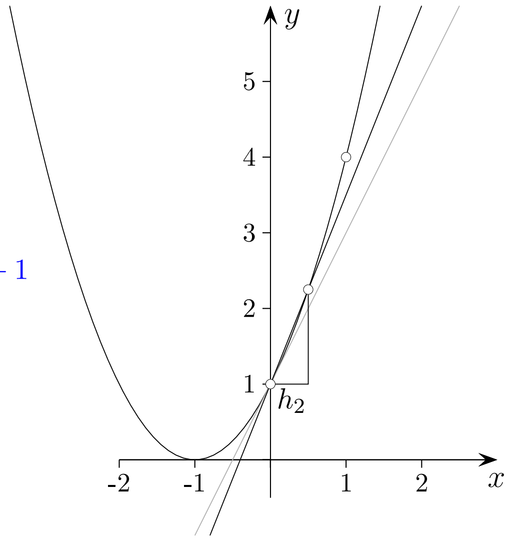
$f(x) = x^2 + 2x + 1$



Punkte auf dem Graphen
$(0|1)$ und $(1|4)$

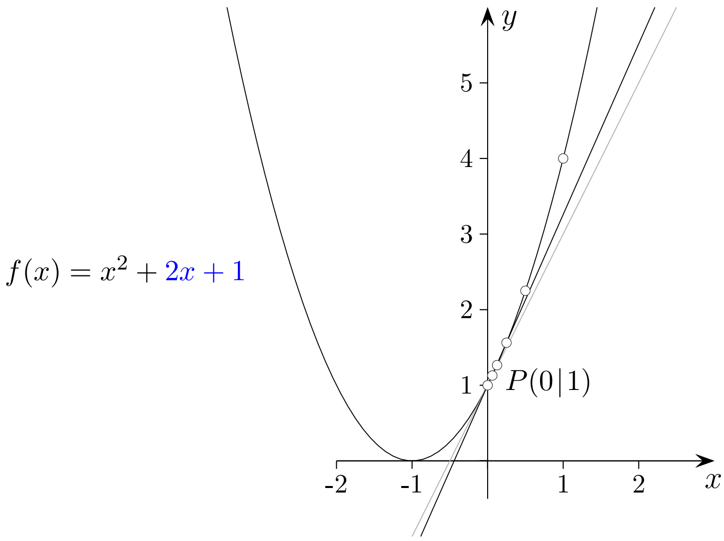
Sekantensteigung
$3 =$ (Differenz der y-Werte)/(Differenz der x-Werte)

$f(x) = x^2 + 2x + 1$



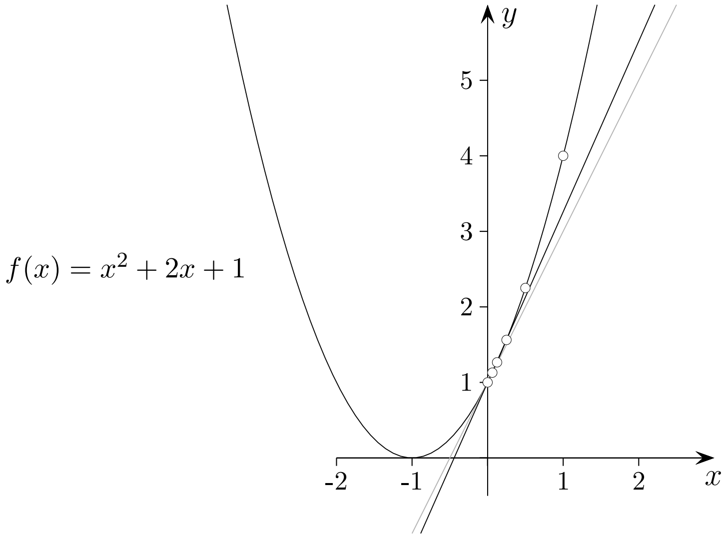
Punkte auf dem Graphen
(0|1) und (0,5|2,25)

Sekantensteigung
2,5



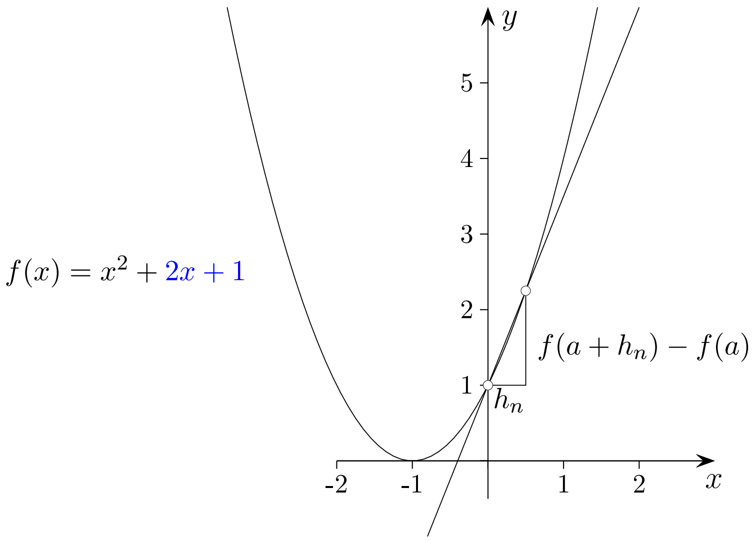
Punkte auf dem Graphen

$(1|4)$, $(0{,}5|2{,}25)$, $(0{,}25|1{,}563)$, $(0{,}125|1{,}266)$, $(0{,}063|1{,}129)$, $(0{,}031|1{,}063)$ $\ldots$



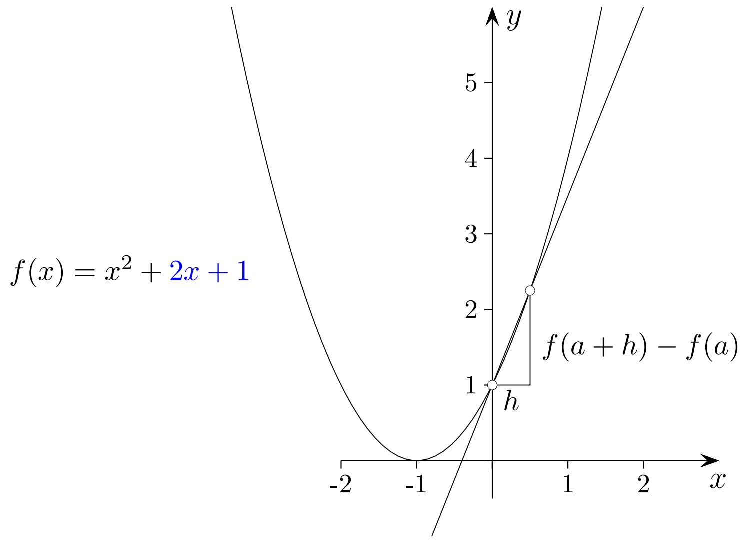
Punkte auf dem Graphen
(1|4), (0,5|2,25), (0,25|1,563), (0,125|1,266), (0,063|1,129), (0,031|1,063) ...

zugehörige Sekantensteigungen
3, 2,5, 2,25, 2,125, 2,0625, 2,0313, 2,0156, 2,0078, 2,0039, 2,00195, 2,00098 ...
strebt gegen 2,00000...



$a = 0$

$$m_{\text{Tangente}} = f'(a) = \lim_{n \to \infty} \frac{f(a + h_n) - f(a)}{h_n}$$



$a = 0$

$$m_{\text{Tangente}} = f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h} = \lim_{h \to 0} \frac{2h + h^2}{h} = \lim_{h \to 0} (2 + h) = 2$$

$$f(x) = \frac{1}{4}x^3 - 2x^2 + 3x + 2$$

Zahl als Summand fällt raus.

Faktorregel: Zahl als Faktor bleibt erhalten.

Potenzregel:
$f(x) = x^n$
$f'(x) = n\,x^{n-1}$

Ableitung (Steigung) $m = 3$

$$f'(x) = \frac{1}{4} \cdot 3x^2 - 2 \cdot 2x + 3$$

Diese Zeile wird übersprungen.

$$f'(x) = \frac{3}{4} \cdot x^2 - 4x + 3$$

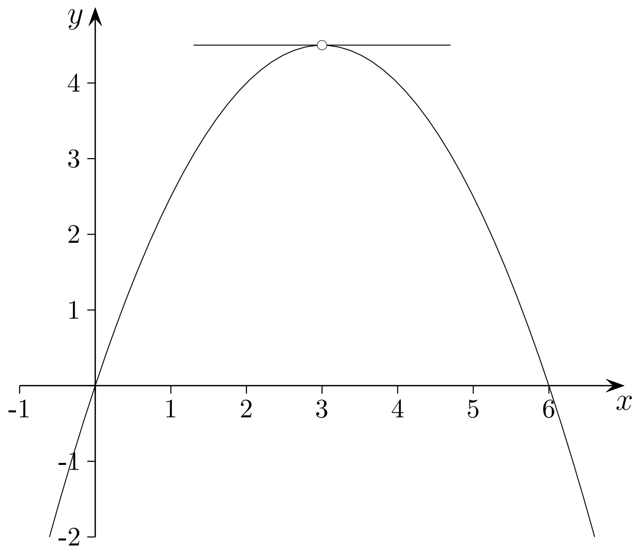
In welchen Punkten besitzt die Funktion waagerechte Tangenten?

a) $f(x) = -\frac{1}{2}x^2 + 3x$

b) $f(x) = -\frac{1}{3}x^3 + 3x^2 - 8x + 7$

Lösungen:

a)
$$\begin{aligned} f(x) &= -\frac{1}{2}x^2 + 3x \\ f'(x) &= -x + 3 \\ 0 &= -x + 3 \\ x &= 3 \qquad\qquad Max\big(3 \mid 4{,}5\big) \end{aligned}$$



b) $$\begin{aligned} f(x) &= -\frac{1}{3}x^3 + 3x^2 - 8x + 7 \\ f'(x) &= -x^2 + 6x - 8 \\ 0 &= -x^2 + 6x - 8 \end{aligned}$$

$$x_1 = 4, \quad x_2 = 2, \quad E_1\big(4 \mid \frac{5}{3}\big), \quad E_2\big(2 \mid \frac{1}{3}\big)$$