Gauß-Verfahren

G.Roolfs

a) $\quad x - 4y - 2z = 4$

b) $\quad 2x - 3y + z = -2$

c) $\quad 3x + y + 2z = 1$

a) $\quad\quad x - 4y - 2z = \quad 4 \mid \cdot(-2) + $ b)

b) $\quad\quad 2x - 3y + \quad z = -2$

c) $\quad\quad \underline{3x + \quad y + 2z = \quad 1}$

a) $\quad\quad x - 4y - 2z = 4 \mid \cdot(-2) + \text{b)} \quad \mid \cdot(-3) + \text{c)}$

b) $\quad\quad 2x - 3y + z = -2$

c) $\quad\quad \underline{3x + y + 2z = 1}$

a) $\qquad x \;-\; 4y \;-\; 2z \;=\;\quad 4 \;\mid\; \cdot(-2) + b) \quad\mid\; \cdot(-3) + c)$

b) $\qquad 2x \;-\; 3y \;+\;\; z \;=\; -2$

c) $\qquad \underline{3x \;+\;\;\; y \;+\; 2z \;=\;\quad 1}$

$\qquad\quad x \;-\; 4y \;-\; 2z \;=\;\quad 4$

a) $\quad x \;-\; 4y \;-\; 2z \;=\; 4 \;\mid\; \cdot(-2) + \text{b)} \quad \mid\; \cdot(-3) + \text{c)}$

b) $\quad 2x \;-\; 3y \;+\; z \;=\; -2$

c) $\quad \underline{3x \;+\; y \;+\; 2z \;=\; 1}$

$\quad\quad x \;-\; 4y \;-\; 2z \;=\; 4$

$\quad\quad\quad\quad\quad 5y \;+\; 5z \;=\; -10$

a) $\quad x - 4y - 2z = 4 \mid \cdot(-2) + b) \quad \mid \cdot(-3) + c)$

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = 1}$

$\qquad x - 4y - 2z = 4$

$\qquad 5y + 5z = -10$

$\qquad \underline{ 13y + 8z = -11}$

Gauß-Verfahren

a) $\quad x - 4y - 2z = \quad 4 \mid \cdot(-2) + b) \qquad \mid \cdot(-3) + c)$

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = \quad 1}$

$\quad\quad x - 4y - 2z = \quad 4$

$\quad\quad\quad\quad 5y + 5z = -10 \mid \cdot(-\frac{13}{5}) + d)$

d) $\quad\quad\quad \underline{13y + 8z = -11}$

a) $\quad x - 4y - 2z = \quad 4 \mid \cdot(-2) + \text{b)} \qquad \mid \cdot(-3) + \text{c)}$

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = \quad 1}$

$\quad\quad x - 4y - 2z = \quad 4$

$\quad\quad\quad\quad 5y + 5z = -10 \mid \cdot(-\frac{13}{5}) + \text{d)}$

d) $\quad\quad \underline{13y + 8z = -11}$

$\quad\quad x - 4y - 2z = \quad 4$

$$
\begin{array}{llrcrl}
\text{a)} & x & - & 4y & - & 2z & = & 4 & | \cdot (-2) + \text{b)} \qquad | \cdot (-3) + \text{c)} \\
\text{b)} & 2x & - & 3y & + & z & = & -2 \\
\text{c)} & \underline{3x} & + & y & + & 2z & = & \underline{1} \\
& x & - & 4y & - & 2z & = & 4 \\
& & & 5y & + & 5z & = & -10 & | \cdot (-\tfrac{13}{5}) + \text{d)} \\
\text{d)} & & & \underline{13y} & + & 8z & = & \underline{-11} \\
& x & - & 4y & - & 2z & = & 4 \\
& & & 5y & + & 5z & = & -10
\end{array}
$$

Gauß-Verfahren

a) $\quad x - 4y - 2z = 4 \mid \cdot(-2) + \text{b}) \qquad \mid \cdot(-3) + \text{c})$

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = 1}$

$\qquad x - 4y - 2z = 4$

$\qquad 5y + 5z = -10 \mid \cdot\left(-\frac{13}{5}\right) + \text{d})$

d) $\qquad \underline{ 13y + 8z = -11}$

$\qquad x - 4y - 2z = 4$

$\qquad 5y + 5z = -10$

$\qquad \underline{ -5z = 15}$

Gauß-Verfahren

$$
\begin{array}{rrrrrrl}
\text{a)} & x & - & 4y & - & 2z & = & 4 \quad | \ \cdot(-2) + \text{b)} \qquad | \ \cdot(-3) + \text{c)} \\
\text{b)} & 2x & - & 3y & + & z & = & -2 \\
\text{c)} & \underline{3x} & + & \underline{y} & + & \underline{2z} & = & \underline{1} \\
\end{array}
$$

$$
\begin{array}{rrrrl}
x & - & 4y & - & 2z & = & 4 \\
 & & 5y & + & 5z & = & -10 \quad | \ \cdot\left(-\frac{13}{5}\right) + \text{d)} \\
\text{d)} & & \underline{13y} & + & \underline{8z} & = & \underline{-11} \\
\end{array}
$$

$$
\begin{array}{rrrrl}
x & - & 4y & - & 2z & = & 4 \\
 & & 5y & + & 5z & = & -10 \\
 & & & - & \underline{5z} & = & \underline{15} \\
 & & & & z & = &
\end{array}
$$

a) $x - 4y - 2z = 4 \mid \cdot(-2) + \text{b)} \qquad \mid \cdot(-3) + \text{c)}$

b) $2x - 3y + z = -2$

c) $\underline{3x + y + 2z = 1}$

$x - 4y - 2z = 4$

$5y + 5z = -10 \mid \cdot(-\frac{13}{5}) + \text{d)}$

d) $\underline{13y + 8z = -11}$

$x - 4y - 2z = 4$

$5y + 5z = -10$

$\underline{- 5z = 15}$

$z = -3$

$y =$

a) $\quad x - 4y - 2z = \quad 4 \mid \cdot(-2) + b) \qquad \mid \cdot(-3) + c)$

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = \quad 1}$

$\qquad x - 4y - 2z = \quad 4$

$\qquad 5y + 5z = -10 \mid \cdot(-\frac{13}{5}) + d)$

d) $\qquad \underline{13y + 8z = -11}$

$\qquad x - 4y - 2z = \quad 4$

$\qquad 5y + 5z = -10$

$\qquad \underline{-5z = \quad 15}$

$\qquad z = -3$

$\qquad y = \quad 1$

$\qquad x =$

Gauß-Verfahren

a) $\quad x - 4y - 2z = 4 \mid \cdot(-2) + $ b) $\qquad \mid \cdot(-3) + $ c)

b) $\quad 2x - 3y + z = -2$

c) $\quad \underline{3x + y + 2z = 1}$

$\qquad x - 4y - 2z = 4$

$\qquad \quad 5y + 5z = -10 \mid \cdot(-\frac{13}{5}) + $ d)

d) $\qquad \quad \underline{13y + 8z = -11}$

$\qquad x - 4y - 2z = 4$

$\qquad \quad 5y + 5z = -10$

$\qquad \qquad \underline{- 5z = 15}$

$\qquad \qquad \quad z = -3$

$\qquad \qquad \quad y = 1$

$\qquad \qquad \quad x = 2$

$$
\begin{array}{rcrcrcr}
 & & 2y & + & z & = & 4 \\
2x & + & 5y & - & 2z & = & -1 \\
x & + & 3y & - & 3z & = & -4
\end{array}
$$

$$2y + z = 4$$
$$2x + 5y - 2z = -1$$
$$x + 3y - 3z = -4$$

Gleichungssystem ordnen

a) $\quad x + 3y - 3z = -4$

b) $\quad 2x + 5y - 2z = -1$

c) $\qquad\quad 2y + z = 4$

a) $x + 3y - 3z = -4$ $| \cdot (-2) + $ b)

b) $2x + 5y - 2z = -1$

c) $2y + z = 4$

a) $x + 3y - 3z = -4$ $| \cdot (-2) + $ b)

b) $2x + 5y - 2z = -1$

c) $ 2y + z = 4$

$x + 3y - 3z = -4$

a)	$x + 3y - 3z = -4$	$\mid \cdot (-2) + $ b)
b)	$2x + 5y - 2z = -1$	
c)	$2y + z = 4$	

$$x + 3y - 3z = -4$$
$$-y + 4z = 7$$

a) $\quad x + 3y - 3z = -4 \qquad | \cdot (-2) + $ b)

b) $\quad 2x + 5y - 2z = -1$

c) $\qquad\qquad 2y + z = 4$

$$\begin{array}{rcl} x + 3y - 3z &=& -4 \\ -y + 4z &=& 7 \\ 2y + z &=& 4 \end{array}$$

$$
\begin{array}{rrrrrll}
\text{a)} & x & + & 3y & - & 3z & = & -4 & \quad | \cdot (-2) + \text{b)} \\
\text{b)} & 2x & + & 5y & - & 2z & = & -1 \\
\text{c)} & & & 2y & + & z & = & 4 \\
\hline
& x & + & 3y & - & 3z & = & -4 \\
& & & -y & + & 4z & = & 7 & \quad | \cdot 2 + \text{d)} \\
\text{d)} & & & 2y & + & z & = & 4 \\
\hline
\end{array}
$$

a) $\quad x + 3y - 3z = -4 \qquad | \cdot (-2) + $ b)

b) $\quad 2x + 5y - 2z = -1$

c) $\qquad\qquad 2y + z = 4$

$$x + 3y - 3z = -4$$
$$-y + 4z = 7 \qquad | \cdot 2 + \text{d)}$$

d) $\qquad\qquad 2y + z = 4$

$$x + 3y - 3z = -4$$

$$
\begin{array}{llrcrcrcrl}
\text{a)} & & x & + & 3y & - & 3z & = & -4 & \quad | \cdot (-2) + \text{b)} \\
\text{b)} & 2x & + & 5y & - & 2z & & = & -1 & \\
\text{c)} & & & & 2y & + & z & = & 4 & \\
\hline
& & x & + & 3y & - & 3z & = & -4 & \\
& & & & -y & + & 4z & = & 7 & \quad | \cdot 2 + \text{d)} \\
\text{d)} & & & & 2y & + & z & = & 4 & \\
\hline
& & x & + & 3y & - & 3z & = & -4 & \\
& & & & -y & + & 4z & = & 7 &
\end{array}
$$

$$
\begin{array}{rrrrrrrl}
\text{a)} & x & + & 3y & - & 3z & = & -4 & \quad | \cdot (-2) + \text{b)} \\
\text{b)} & 2x & + & 5y & - & 2z & = & -1 \\
\text{c)} & & & 2y & + & z & = & 4 \\
\hline
& x & + & 3y & - & 3z & = & -4 \\
& & & -y & + & 4z & = & 7 & \quad | \cdot 2 + \text{d)} \\
\text{d)} & & & 2y & + & z & = & 4 \\
\hline
& x & + & 3y & - & 3z & = & -4 \\
& & & -y & + & 4z & = & 7 \\
& & & & + & 9z & = & 18 \\
\hline
& & & & & z & = &
\end{array}
$$

a) $\quad x + 3y - 3z = -4 \quad | \cdot (-2) + $ b)

b) $\quad 2x + 5y - 2z = -1$

c) $\qquad\quad 2y + z = 4$

$\overline{ x + 3y - 3z = -4}$

$\qquad\quad -y + 4z = 7 \quad | \cdot 2 + $ d)

d) $\qquad\qquad 2y + z = 4$

$\overline{ x + 3y - 3z = -4}$

$\qquad\quad -y + 4z = 7$

$\qquad\qquad\quad + 9z = 18$

$\overline{\qquad\qquad\qquad z = 2}$

$\qquad\qquad\qquad y =$

a) $\quad x + 3y - 3z = -4 \quad | \cdot (-2) + b)$

b) $\quad 2x + 5y - 2z = -1$

c) $\qquad\quad 2y + z = 4$

$\overline{\quad x + 3y - 3z = -4 \qquad\qquad\qquad}$

$\qquad\quad -y + 4z = 7 \quad | \cdot 2 + d)$

d) $\qquad\quad 2y + z = 4$

$\overline{\quad x + 3y - 3z = -4 \qquad\qquad\qquad}$

$\qquad\quad -y + 4z = 7$

$\qquad\qquad\; + 9z = 18$

$\overline{\qquad\qquad\quad z = 2 \qquad\qquad\quad}$

$\qquad\qquad\quad y = 1$

$\qquad\qquad\quad x =$

$$
\begin{array}{lrcrcrcrl}
\text{a)} & x & + & 3y & - & 3z & = & -4 & \quad |\ \cdot(-2) + \text{b)} \\
\text{b)} & 2x & + & 5y & - & 2z & = & -1 & \\
\text{c)} & & & 2y & + & z & = & 4 & \\
\hline
& x & + & 3y & - & 3z & = & -4 & \\
& & & -y & + & 4z & = & 7 & \quad |\ \cdot 2 + \text{d)} \\
\text{d)} & & & 2y & + & z & = & 4 & \\
\hline
& x & + & 3y & - & 3z & = & -4 & \\
& & & -y & + & 4z & = & 7 & \\
& & & & & 9z & = & 18 & \\
\hline
& & & & & z & = & 2 & \\
& & & & & y & = & 1 & \\
& & & & & x & = & -1 &
\end{array}
$$

a) $\quad x - 2y + 3z = 4$

b) $\quad 3x + y - 5z = 5$

c) $\quad 2x - 3y + 4z = 7$

a) $\quad\quad x - 2y + 3z = 4 \quad | \cdot (-3) + \text{b)}$

b) $\quad\quad 3x + y - 5z = 5$

c) $\quad\quad \underline{2x - 3y + 4z = 7}$

a) $\qquad x - 2y + 3z = 4 \qquad | \cdot (-3) + \text{b}) \qquad | \cdot (-2) + \text{c})$

b) $\qquad 3x + y - 5z = 5$

c) $\qquad \underline{2x - 3y + 4z = 7}$

a)	$x - 2y + 3z = 4$	$\mid \cdot (-3) + $ b)	$\mid \cdot (-2) + $ c)
b)	$3x + y - 5z = 5$		
c)	$2x - 3y + 4z = 7$		
	$x - 2y + 3z = 4$		

a) $\quad\quad x - 2y + 3z = 4 \quad | \cdot (-3) + $ b) $\quad | \cdot (-2) + $ c)

b) $\quad\quad 3x + y - 5z = 5$

c) $\quad\quad \underline{2x - 3y + 4z = 7}$

$\quad\quad\quad x - 2y + 3z = 4$

$\quad\quad\quad\quad\quad 7y - 14z = -7$

a) $\quad x - 2y + 3z = 4 \qquad | \cdot (-3) + \text{b)} \qquad | \cdot (-2) + \text{c)}$

b) $\quad 3x + y - 5z = 5$

c) $\quad \underline{2x - 3y + 4z = 7}$

$\quad x - 2y + 3z = 4$

$\quad 7y - 14z = -7$

$\quad \underline{y - 2z = -1}$

a) $\qquad x - 2y + 3z = 4 \qquad | \cdot (-3) + \text{b)} \qquad | \cdot (-2) + \text{c)}$

b) $\qquad 3x + y - 5z = 5$

c) $\qquad \dfrac{2x - 3y + 4z = 7}{}$

$\qquad\qquad x - 2y + 3z = 4$

$\qquad\qquad 7y - 14z = -7 \qquad | : 7$

d) $\qquad \dfrac{\qquad\quad y - 2z = -1}{}$

a) $\quad x - 2y + 3z = 4$

b) $\quad 3x + y - 5z = 5$

c) $\quad 2x - 3y + 4z = 7$

$$\overline{\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 4 \\ & & 7y & - & 14z & = & -7 \end{array}} \quad | : 7$$

d) $\qquad\qquad y - 2z = -1$

$$\overline{\begin{array}{rcrcrcr} x & - & 2y & + & 3z & = & 4 \\ & & y & - & 2z & = & -1 \\ & & y & - & 2z & = & -1 \end{array}}$$

$$
\begin{array}{lrcrcrcl}
\text{a)} & x & - & 2y & + & 3z & = & 4 \\
\text{b)} & 3x & + & y & - & 5z & = & 5 \\
\text{c)} & 2x & - & 3y & + & 4z & = & 7 \\
\hline
& x & - & 2y & + & 3z & = & 4 \\
& & & 7y & - & 14z & = & -7 \quad |\,:7 \\
\text{d)} & & & y & - & 2z & = & -1 \\
\hline
& x & - & 2y & + & 3z & = & 4 \\
& & & y & - & 2z & = & -1 \quad |\,\cdot(-1)+\text{e)} \\
\text{e)} & & & y & - & 2z & = & -1 \\
\hline
\end{array}
$$

a) $\quad x - 2y + 3z = 4$

b) $\quad 3x + y - 5z = 5$

c) $\quad \underline{2x - 3y + 4z = 7}$

$\quad x - 2y + 3z = 4$

$\qquad 7y - 14z = -7 \quad | : 7$

d) $\qquad \underline{y - 2z = -1}$

$\quad x - 2y + 3z = 4$

$\qquad y - 2z = -1 \quad | \cdot (-1) + e)$

e) $\qquad \underline{y - 2z = -1}$

$\quad x - 2y + 3z = 4$

$\qquad y - 2z = -1$

$\qquad\qquad 0 = 0$

a)	$x - 2y + 3z = 4$	
b)	$3x + y - 5z = 5$	
c)	$2x - 3y + 4z = 7$	

$$x - 2y + 3z = 4$$
$$7y - 14z = -7 \quad | : 7$$

d)
$$y - 2z = -1$$

$$x - 2y + 3z = 4$$
$$y - 2z = -1 \quad | \cdot (-1) + \text{e})$$

e)
$$y - 2z = -1$$

$$x - 2y + 3z = 4$$
$$y - 2z = -1$$
$$0 = 0$$

2 Gleichungen, 3 Unbekannte:

a) $\qquad x - 2y + 3z = 4$

b) $\qquad 3x + y - 5z = 5$

c) $\qquad \dfrac{2x - 3y + 4z = 7}{}$

$\qquad\qquad x - 2y + 3z = 4$

$\qquad\qquad 7y - 14z = -7 \qquad | : 7$

d) $\qquad\qquad\quad y - 2z = -1$

$\qquad\qquad \dfrac{}{x - 2y + 3z = 4}$

$\qquad\qquad\quad y - 2z = -1 \qquad | \cdot (-1) + e)$

e) $\qquad\qquad\quad y - 2z = -1$

$\qquad\qquad \dfrac{}{x - 2y + 3z = 4}$

$\qquad\qquad\quad y - 2z = -1$

$\qquad\qquad\qquad\quad 0 = 0$

2 Gleichungen, 3 Unbekannte: Es gibt unendlich viele Lösungen.

$$
\begin{aligned}
x \; - \; 2y \; + \; 3z &= 4 \\
y \; - \; 2z &= -1 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
x \; - \; 2y \; + \; 3z \; &= \; 4 \\
y \; - \; 2z \; &= \; -1 \\
0 \; &= \; 0 \\
\hline
z \; &= \; k
\end{aligned}
$$

$$
\begin{aligned}
x \;-\; 2y \;+\; 3z &= 4 \\
y \;-\; 2z &= -1 \\
0 &= 0 \\
\hline
z &= k \\
y &=
\end{aligned}
$$

$$\begin{aligned}
x \;-\; 2y \;+\; 3z \;&=\; 4 \\
y \;-\; 2z \;&=\; -1 \\
0 \;&=\; 0 \\
\hline
z \;&=\; k \\
y \;&=\; -1 + 2k
\end{aligned}$$

$$
\begin{aligned}
x \;-\; 2y \;+\; 3z &= 4 \\
y \;-\; 2z &= -1 \\
0 &= 0 \\
\hline
z &= k \\
y &= -1 + 2k \\
x &=
\end{aligned}
$$

$$
\begin{aligned}
x \ - \ 2y \ + \ 3z &= 4 \\
y \ - \ 2z &= -1 \\
0 &= 0
\end{aligned}
$$

$$
\begin{aligned}
z &= k \\
y &= -1 + 2k \\
x &= 2 + k
\end{aligned}
$$

$$
\begin{aligned}
x \ - \ 2y \ + \ 3z \ &= \ 4 \\
y \ - \ 2z \ &= \ -1 \\
\hline
0 \ &= \ 0 \\
\hline
z \ &= \ k \\
y \ &= \ -1 + 2k \\
x \ &= \ 2 + k
\end{aligned}
$$

$$
\begin{pmatrix} 2 + k \\ -1 + 2k \\ k \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 0 \end{pmatrix} + k \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}, \quad k \in \mathbb{R}
$$