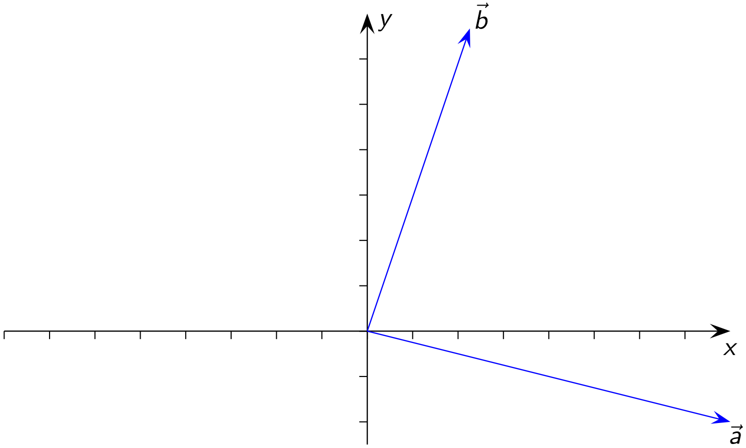
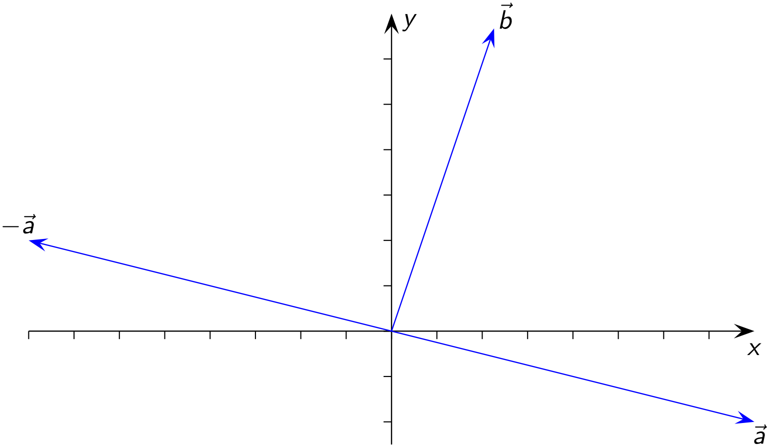
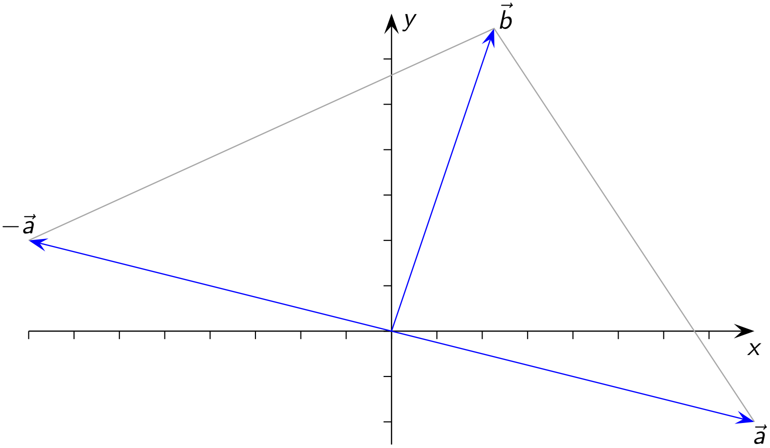


y
$\vec{b}$
x
$\vec{a}$

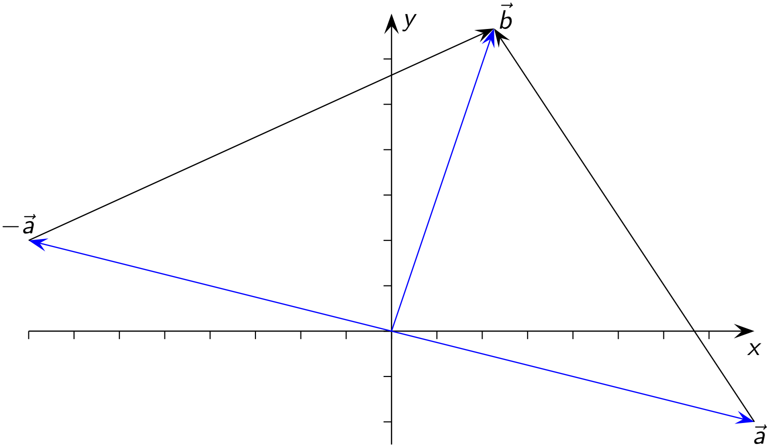
Kriterium für $\vec{a} \perp \vec{b}$

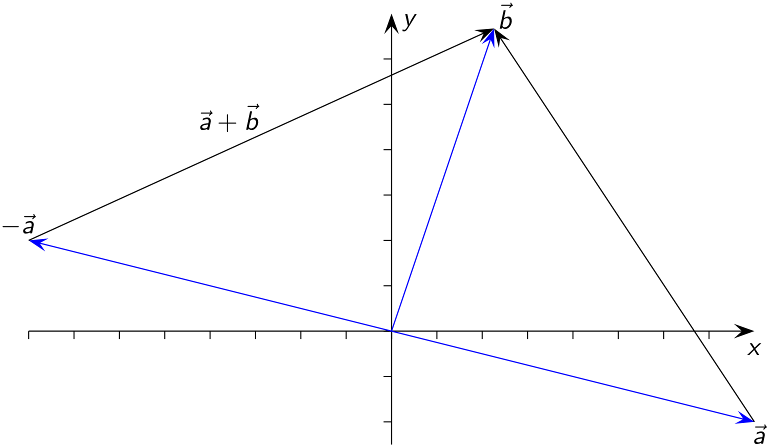


Kriterium für $\vec{a} \perp \vec{b}$



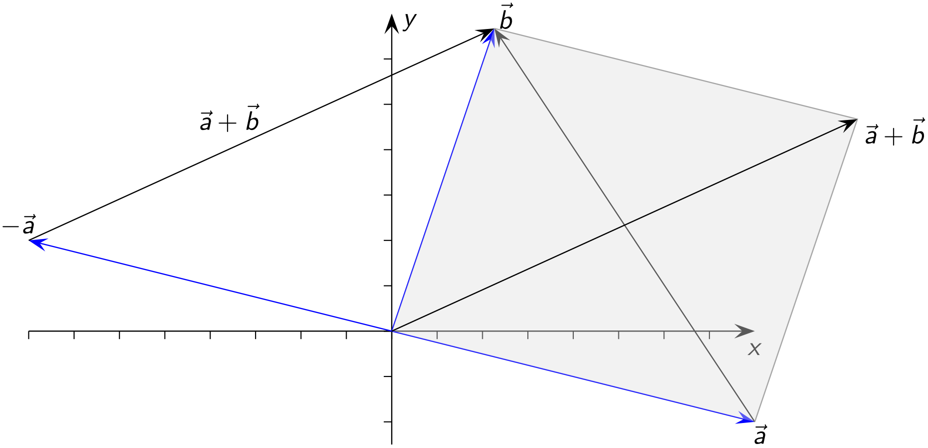
Kriterium für $\vec{a} \perp \vec{b}$

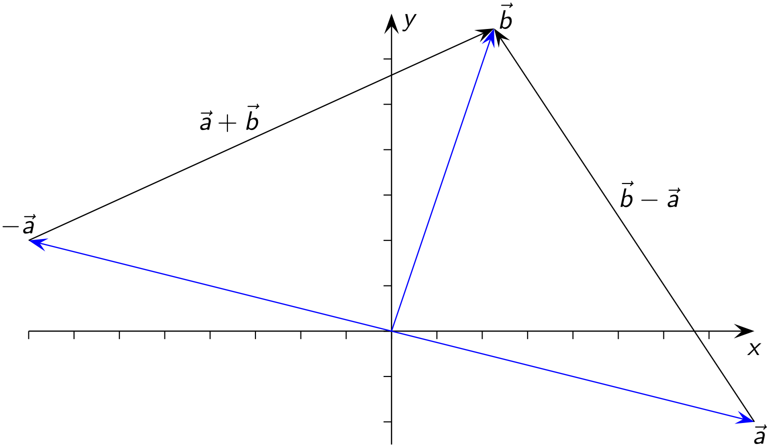




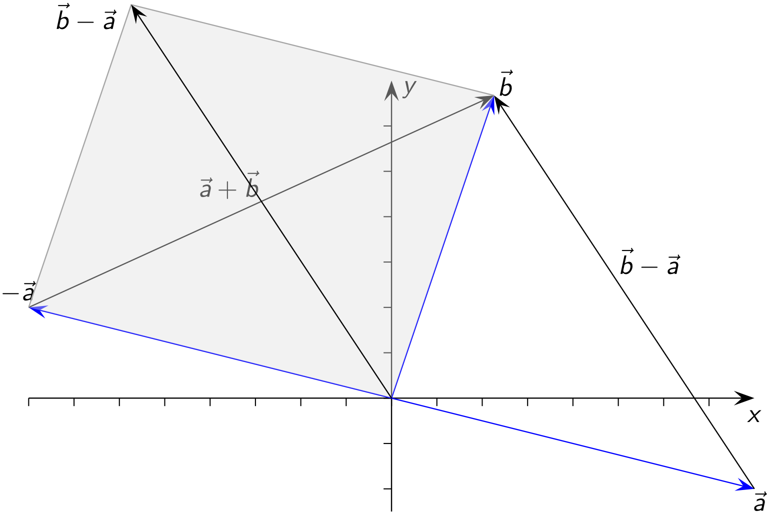
y
$\vec{b}$
$\vec{a}+\vec{b}$
$-\vec{a}$
x
$\vec{a}$

Kriterium für $\vec{a} \perp \vec{b}$

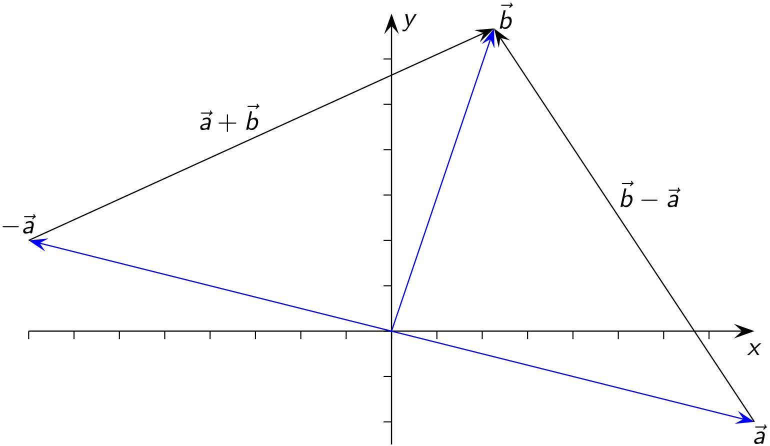




y
$\vec{b}$
$\vec{a} + \vec{b}$
$\vec{b} - \vec{a}$
$-\vec{a}$
x
$\vec{a}$


$\vec{b} - \vec{a}$
y
$\vec{b}$
$\vec{a} + \vec{b}$
$\vec{b} - \vec{a}$
$-\vec{a}$
x
$\vec{a}$

Kriterium für $\vec{a} \perp \vec{b}$



$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

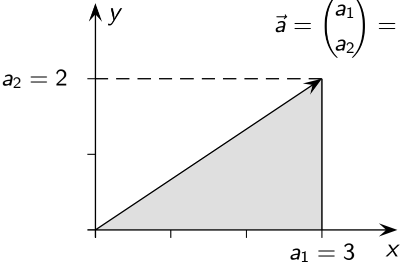
$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Länge: $\quad |\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Länge: $\quad |\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$



$$|\,\vec{a}\,| = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Länge: $|\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Länge: $|\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

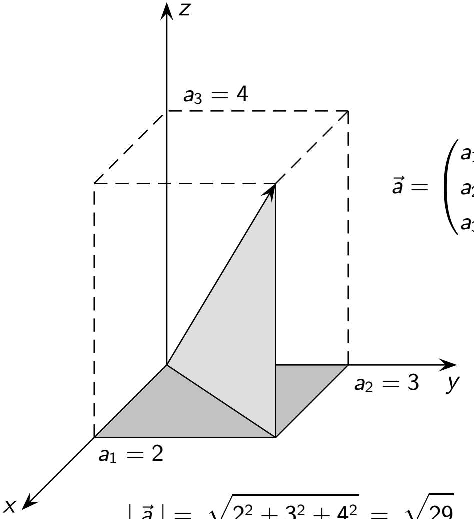
Länge: $|\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix}$$

Länge: $\quad |\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2}$

$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$$

Länge: $\quad |\,\vec{a}\,| = \sqrt{a_1^2 + a_2^2 + a_3^2}$



$$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 4 \end{pmatrix}$$

$$|\,\vec{a}\,| = \sqrt{2^2 + 3^2 + 4^2} = \sqrt{29}$$

Kriterium für $\vec{a} \perp \vec{b}$

$\vec{a} \perp \vec{b}$

$$\vec{a} \perp \vec{b} \quad \Longleftrightarrow$$

Kriterium für $\vec{a} \perp \vec{b}$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| =$$

$$\vec{a} \perp \vec{b} \quad \Longleftrightarrow \quad |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$
$$\iff \sqrt{}$$

Kriterium für $\vec{a} \perp \vec{b}$

$$\begin{aligned} \vec{a} \perp \vec{b} \quad &\Longleftrightarrow \quad |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,| \\ &\Longleftrightarrow \quad \sqrt{(a_1 + b_1)^2 \qquad\qquad\qquad\qquad} \end{aligned}$$

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$
$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} =$$

$$\vec{a} + \vec{b} = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = \begin{pmatrix} a_1 + b_1 \\ a_2 + b_2 \end{pmatrix}$$

Kriterium für $\vec{a} \perp \vec{b}$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2}$$

$$\vec{b} - \vec{a} = \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} - \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_1 - a_1 \\ b_2 - a_2 \end{pmatrix}$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ \ )^2$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ )^2$$

$$\iff$$

Kriterium für $\vec{a} \perp \vec{b}$

$$\vec{a} \perp \vec{b} \iff |\, \vec{a} + \vec{b} \,| = |\, \vec{b} - \vec{a} \,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\, (\ )^2$$

$$\iff (a_1 + b_1)^2$$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \quad &\Longleftrightarrow \quad |\, \vec{a} + \vec{b} \,| = |\, \vec{b} - \vec{a} \,| \\
&\Longleftrightarrow \quad \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} \;=\; \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ \ )^2 \\
&\Longleftrightarrow \quad (a_1 + b_1)^2 + (a_2 + b_2)^2 \;=
\end{aligned}
$$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \quad &\Longleftrightarrow \quad |\, \vec{a} + \vec{b} \,| = |\, \vec{b} - \vec{a} \,| \\
&\Longleftrightarrow \quad \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\; (\;\;)^2 \\
&\Longleftrightarrow \quad (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2
\end{aligned}
$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ )^2$$

$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\ \text{binomische Formeln}$$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \quad &\iff \quad |\, \vec{a} + \vec{b} \,| = |\, \vec{b} - \vec{a} \,| \\
&\iff \quad \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} \;=\; \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\;)^2 \\
&\iff \quad (a_1 + b_1)^2 + (a_2 + b_2)^2 \;=\; (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\,\text{binomische Formeln} \\
&\iff
\end{aligned}
$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\;(\;)^2$$

$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\; \text{binomische Formeln}$$

$$\iff 2a_1 b_1$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ \ )^2$$

$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\text{ binomische Formeln}$$

$$\iff 2a_1 b_1 + 2a_2 b_2 =$$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \quad &\Longleftrightarrow \quad |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,| \\
&\Longleftrightarrow \quad \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} \;=\; \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\;(\;\;)^2 \\
&\Longleftrightarrow \quad (a_1 + b_1)^2 + (a_2 + b_2)^2 \;=\; (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\;\text{binomische Formeln} \\
&\Longleftrightarrow \quad 2a_1 b_1 + 2a_2 b_2 \;=\; -\,2a_1 b_1
\end{aligned}
$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\,(\ )^2$$

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$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\ (\ )^2$$

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$$\iff 2a_1 b_1 + 2a_2 b_2 = -2a_1 b_1 - 2a_2 b_2 \qquad |\ \text{Quadrate fallen heraus}$$

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$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\; \text{binomische Formeln}$$

$$\iff 2a_1 b_1 + 2a_2 b_2 = -\,2a_1 b_1 - 2a_2 b_2 \qquad |\; \text{Quadrate fallen heraus}$$

$$\iff$$

$$\vec{a} \perp \vec{b} \iff |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,|$$

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$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\ \text{binomische Formeln}$$

$$\iff 2a_1b_1 + 2a_2b_2 = -2a_1b_1 - 2a_2b_2 \qquad |\ \text{Quadrate fallen heraus}$$

$$\iff 4a_1b_1 + 4a_2b_2 = 0$$

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$$\iff \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} \qquad |\ (\ )^2$$

$$\iff (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad |\ \text{binomische Formeln}$$

$$\iff 2a_1 b_1 + 2a_2 b_2 = -2a_1 b_1 - 2a_2 b_2 \qquad |\ \text{Quadrate fallen heraus}$$

$$\iff 4a_1 b_1 + 4a_2 b_2 = 0$$

$$\iff$$

$$\begin{aligned}
\vec{a} \perp \vec{b} \iff & \, |\, \vec{a} + \vec{b} \,| = |\, \vec{b} - \vec{a} \,| \\
\iff & \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} && |\;(\;\;)^2 \\
\iff & (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 && |\;\text{binomische Formeln} \\
\iff & 2a_1 b_1 + 2a_2 b_2 = -2a_1 b_1 - 2a_2 b_2 && |\;\text{Quadrate fallen heraus} \\
\iff & 4a_1 b_1 + 4a_2 b_2 = 0 \\
\iff & a_1 b_1 + a_2 b_2 = 0
\end{aligned}$$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \quad &\Longleftrightarrow \quad |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,| \\
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&\Longleftrightarrow \quad (a_1 + b_1)^2 + (a_2 + b_2)^2 \;=\; (b_1 - a_1)^2 + (b_2 - a_2)^2 \qquad \Big|\; \text{binomische Formeln} \\
&\Longleftrightarrow \quad 2a_1b_1 + 2a_2b_2 \;=\; -\,2a_1b_1 - 2a_2b_2 \qquad \Big|\; \text{Quadrate fallen heraus} \\
&\Longleftrightarrow \quad 4a_1b_1 + 4a_2b_2 \;=\; 0 \\
&\Longleftrightarrow \quad a_1b_1 + a_2b_2 = 0
\end{aligned}
$$

Definition des Skalarprodukts (Ebene): $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} =$

$$
\begin{aligned}
\vec{a} \perp \vec{b} \iff & \ |\,\vec{a} + \vec{b}\,| = |\,\vec{b} - \vec{a}\,| \\
\iff & \sqrt{(a_1 + b_1)^2 + (a_2 + b_2)^2} = \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2} && |\ (\ )^2 \\
\iff & (a_1 + b_1)^2 + (a_2 + b_2)^2 = (b_1 - a_1)^2 + (b_2 - a_2)^2 && |\ \text{binomische Formeln} \\
\iff & 2a_1 b_1 + 2a_2 b_2 = -2a_1 b_1 - 2a_2 b_2 && |\ \text{Quadrate fallen heraus} \\
\iff & 4a_1 b_1 + 4a_2 b_2 = 0 \\
\iff & a_1 b_1 + a_2 b_2 = 0
\end{aligned}
$$

Definition des Skalarprodukts (Ebene): $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = a_1 b_1 + a_2 b_2$

Definition des Skalarprodukts (Raum):

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} =$$

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Beispiel:

$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} =$$

Definition des Skalarprodukts (Raum):

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Beispiel:

$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = 1 \cdot 0$$

Definition des Skalarprodukts (Raum):

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Beispiel:

$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = 1 \cdot 0 + (-3) \cdot 4 +$$

Definition des Skalarprodukts (Raum):

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

Beispiel:

$$\begin{pmatrix} 1 \\ -3 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 4 \\ 6 \end{pmatrix} = 1 \cdot 0 + (-3) \cdot 4 + 2 \cdot 6 =$$

Definition des Skalarprodukts (Raum):

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \cdot \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

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Beispiel:

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Die Vektoren stehen senkrecht aufeinander.

Definition des Skalarprodukts (Raum):

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Beispiel:

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