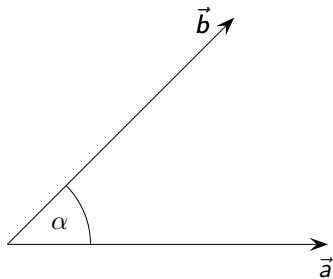
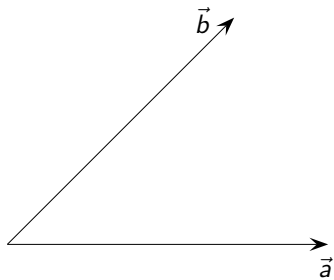
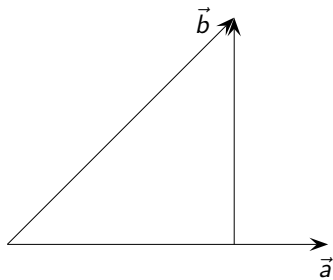


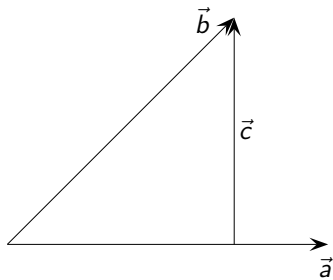
Mit dem Skalarprodukt $\vec{a} \cdot \vec{b}$ kann der Winkel berechnet werden, den zwei Vektoren miteinander einschließen.

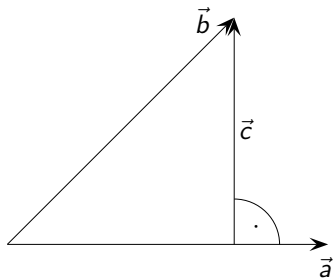


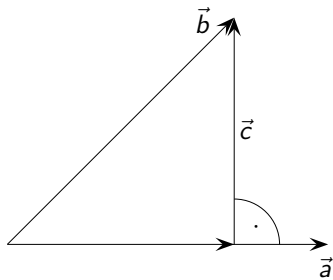
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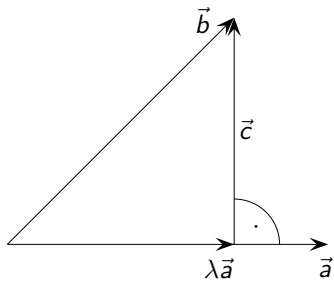


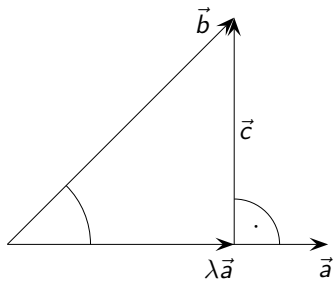


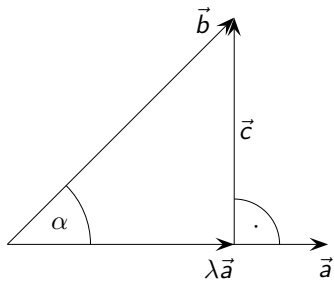


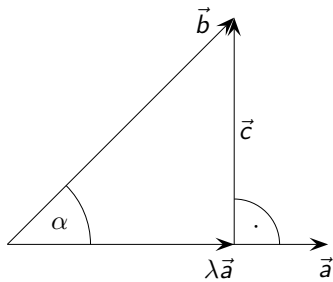




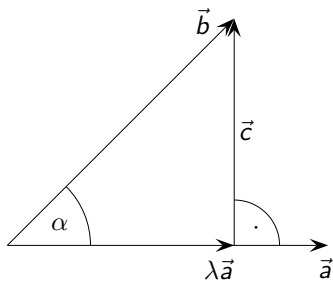




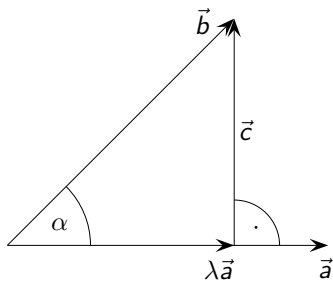




$$\cos \alpha =$$

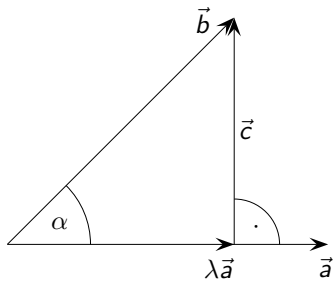


$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

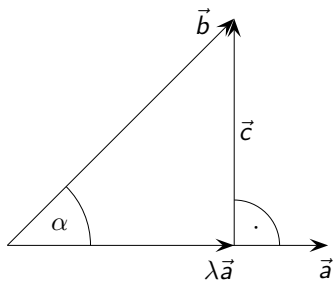
für positives λ



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

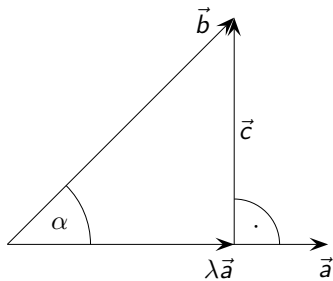
$$\vec{a} \cdot \vec{b} =$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

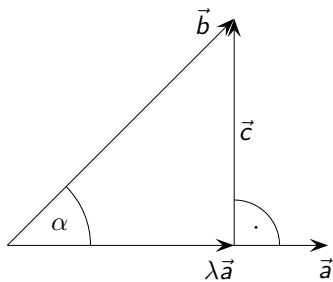
$$\vec{a} \cdot \vec{b} = \vec{a} \cdot \lambda \vec{a}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

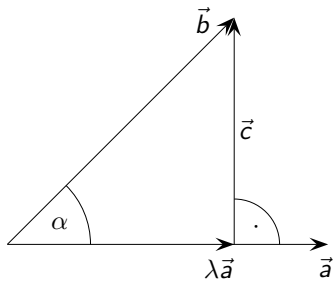
$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

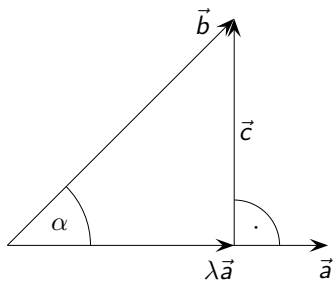
$$\begin{aligned}\vec{a} \cdot \vec{b} &= \vec{a} \cdot (\lambda\vec{a} + \vec{c}) \\ &= \end{aligned}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

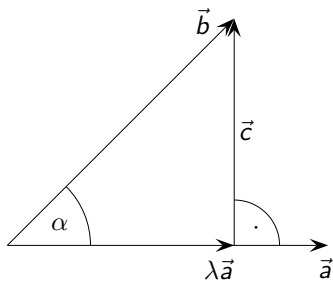
$$\begin{aligned}\vec{a} \cdot \vec{b} &= \vec{a} \cdot (\lambda \vec{a} + \vec{c}) \\ &= \lambda \vec{a}^2\end{aligned}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \vec{a} \cdot (\lambda \vec{a} + \vec{c}) \\ &= \lambda \vec{a}^2 \end{aligned}$$

$$\vec{a} \cdot \vec{c} = 0$$



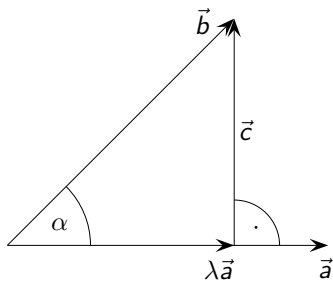
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$$= \lambda \vec{a}^2$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda =$$



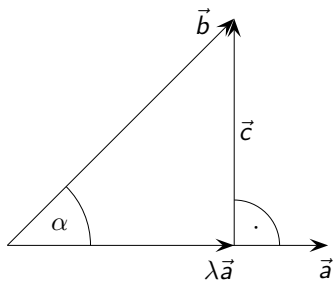
$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$

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$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

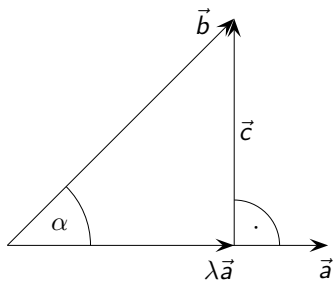
$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$

$$= \lambda \vec{a}^2$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

$$|\vec{a}| =$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

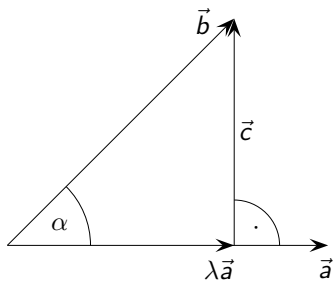
$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$

$$= \lambda \vec{a}^2$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

$$|\vec{a}| = \sqrt{\vec{a}^2}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

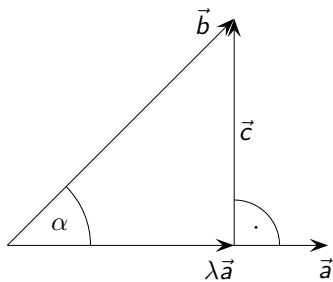
$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$

$$= \lambda \vec{a}^2$$

$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

$$|\vec{a}| = \sqrt{\vec{a}^2} \implies$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

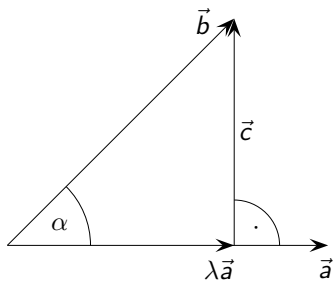
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für positives λ

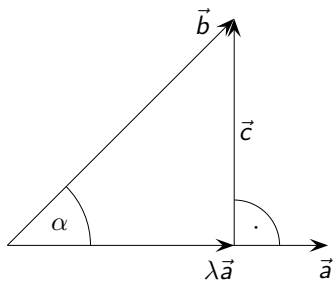
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für positives λ

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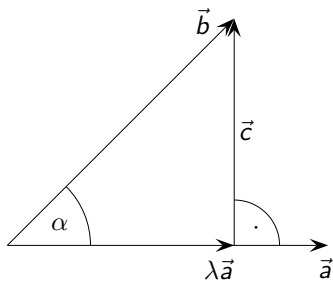
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$$\cos \alpha =$$



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für positives λ

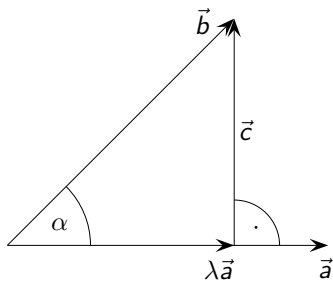
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$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

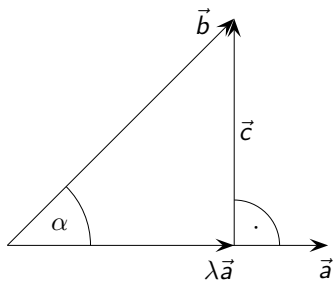
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$$|\vec{a}| = \sqrt{\vec{a}^2} \implies |\vec{a}|^2 = \vec{a}^2$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \text{oder}$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|}$$

für positives λ

$$\vec{a} \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{a} + \vec{c})$$

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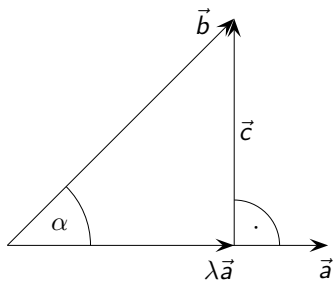
$$\vec{a} \cdot \vec{c} = 0$$

$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

$$|\vec{a}| = \sqrt{\vec{a}^2} \implies |\vec{a}|^2 = \vec{a}^2$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

oder $\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \vec{a} \cdot (\lambda \vec{a} + \vec{c}) \\ &= \lambda \vec{a}^2 \end{aligned}$$

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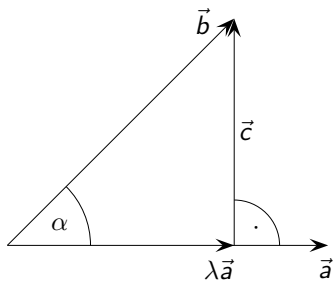
$$\lambda = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2}$$

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$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|}$$

$$\text{oder } \cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\vec{a} \cdot \vec{b} =$$



$$\cos \alpha = \frac{\lambda |\vec{a}|}{|\vec{b}|} \quad \text{für positives } \lambda$$

$$\begin{aligned} \vec{a} \cdot \vec{b} &= \vec{a} \cdot (\lambda \vec{a} + \vec{c}) \\ &= \lambda \vec{a}^2 \end{aligned}$$

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$$|\vec{a}| = \sqrt{\vec{a}^2} \implies |\vec{a}|^2 = \vec{a}^2$$

$$\cos \alpha = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| |\vec{b}|} \quad \text{oder} \quad \cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos \alpha$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha =$$

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Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ =$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix},$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ =$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ = \frac{1}{21} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ = \frac{1}{21} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\alpha =$$

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Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

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$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ = \frac{1}{21} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\alpha = \arccos\left(\frac{9}{14 \cdot 21}\right)$$

$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

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$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ = \frac{1}{21} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\alpha = \arccos\left(\frac{9}{14 \cdot 21}\right)$$

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$$\vec{a} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

Winkel, den die Ortsvektoren einschließen?

$$\cos \alpha = \vec{a}^\circ \cdot \vec{b}^\circ$$

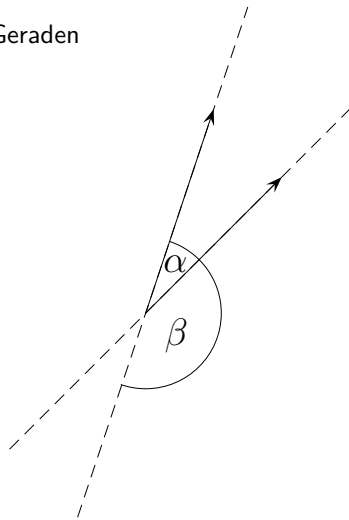
$$\alpha = \arccos(\vec{a}^\circ \cdot \vec{b}^\circ)$$

$$\vec{a}^\circ = \frac{1}{14} \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}, \quad \vec{b}^\circ = \frac{1}{21} \begin{pmatrix} -2 \\ 1 \\ 4 \end{pmatrix}$$

$$\alpha = \arccos\left(\frac{9}{14 \cdot 21}\right)$$

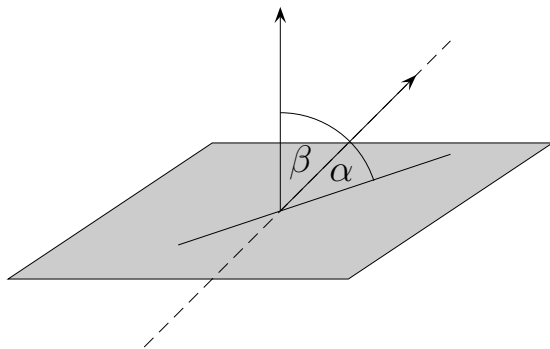
$$\alpha = 58,3^\circ$$

Der Schnittwinkel zweier sich schneidender Geraden ist der kleinere der beiden Winkel α und β .



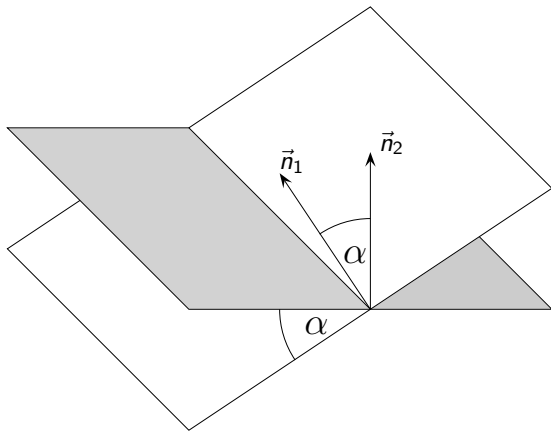
$\cos \alpha = |\vec{a}^\circ \cdot \vec{b}^\circ|$ führt stets zum (korrekten) spitzen Winkel.

Der Schnittwinkel α von Gerade und Ebene ist der Winkel zwischen der Geraden und ihrer senkrechten Projektion.

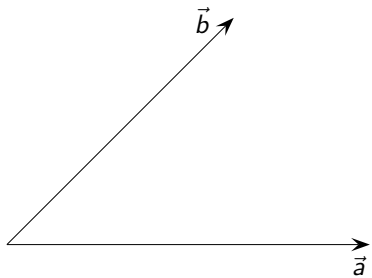


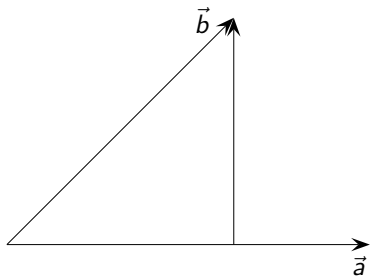
$$\sin \alpha = |\vec{a}^\circ \cdot \vec{b}^\circ|$$

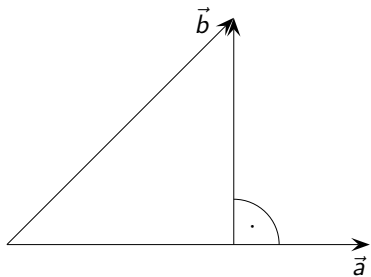
Der Schnittwinkel α von zwei Ebenen wird durch die Normalenvektoren bestimmt.

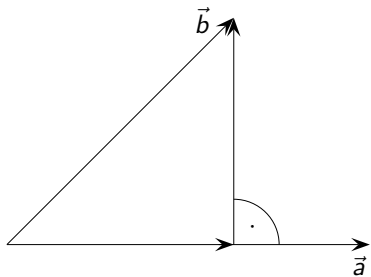


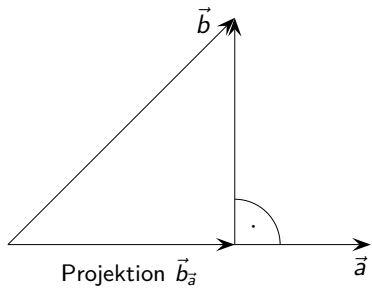
$$\cos \alpha = |\vec{n}_1^o \cdot \vec{n}_2^o|$$

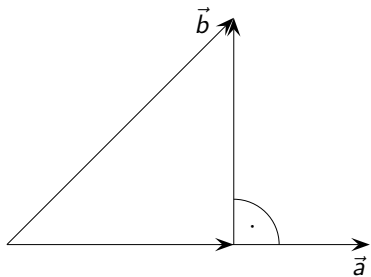


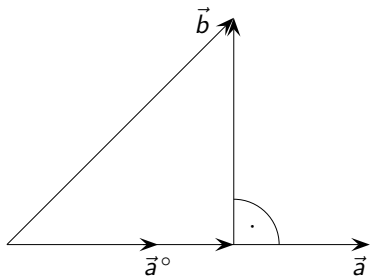


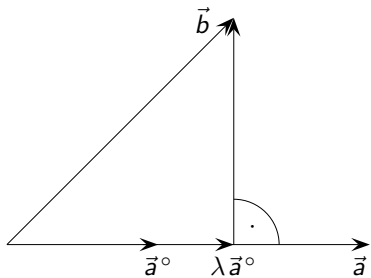


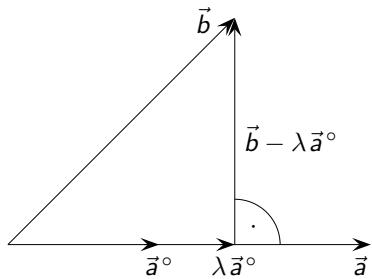


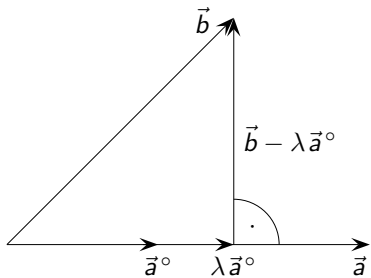




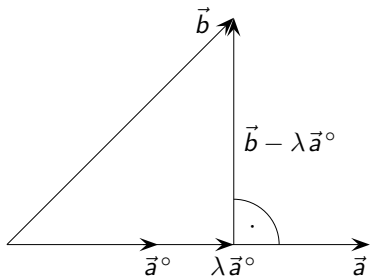






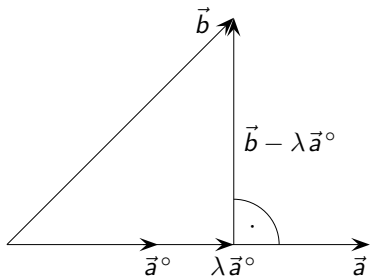


$$\vec{a}^\circ \perp (\vec{b} - \lambda \vec{a}^\circ)$$



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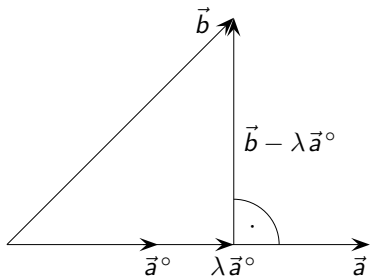
$$0 = \vec{a}^\circ \cdot (\vec{b} - \lambda \vec{a}^\circ)$$



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$$= \vec{a}^\circ \cdot \vec{b} - \lambda$$

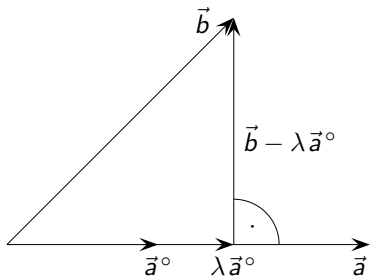


$$\vec{a}^\circ \perp (\vec{b} - \lambda \vec{a}^\circ)$$

$$0 = \vec{a}^\circ \cdot (\vec{b} - \lambda \vec{a}^\circ)$$

$$= \vec{a}^\circ \cdot \vec{b} - \lambda$$

$$\vec{a}^\circ \cdot \vec{a}^\circ = 0$$



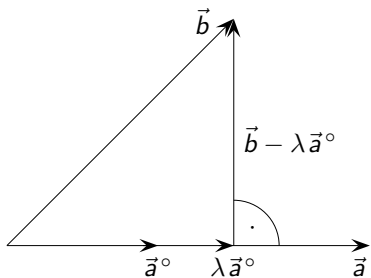
$$\vec{a}^\circ \perp (\vec{b} - \lambda \vec{a}^\circ)$$

$$0 = \vec{a}^\circ \cdot (\vec{b} - \lambda \vec{a}^\circ)$$

$$= \vec{a}^\circ \cdot \vec{b} - \lambda$$

$$\lambda = \vec{a}^\circ \cdot \vec{b}$$

$$\vec{a}^\circ \cdot \vec{a}^\circ = 0$$



$$\vec{a}^\circ \perp (\vec{b} - \lambda \vec{a}^\circ)$$

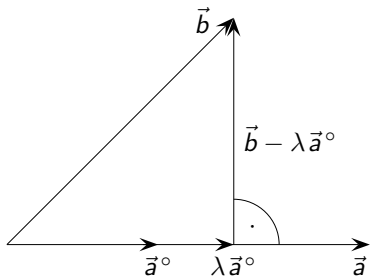
$$0 = \vec{a}^\circ \cdot (\vec{b} - \lambda \vec{a}^\circ)$$

$$= \vec{a}^\circ \cdot \vec{b} - \lambda$$

$$\vec{a}^\circ \cdot \vec{a}^\circ = 0$$

$$\lambda = \vec{a}^\circ \cdot \vec{b}$$

$$\vec{b}_{\vec{a}} =$$



$$\vec{a}^\circ \perp (\vec{b} - \lambda \vec{a}^\circ)$$

$$0 = \vec{a}^\circ \cdot (\vec{b} - \lambda \vec{a}^\circ)$$

$$= \vec{a}^\circ \cdot \vec{b} - \lambda$$

$$\vec{a}^\circ \cdot \vec{a}^\circ = 0$$

$$\lambda = \vec{a}^\circ \cdot \vec{b}$$

$$\vec{b}_{\vec{a}} = (\vec{a}^\circ \cdot \vec{b}) \vec{a}^\circ$$