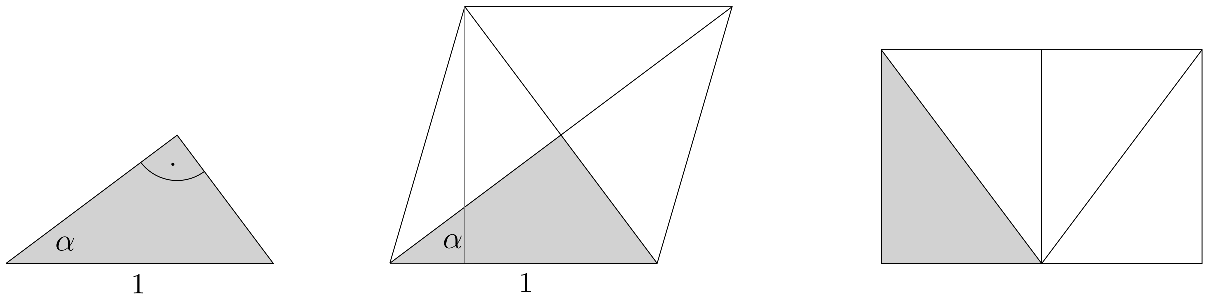
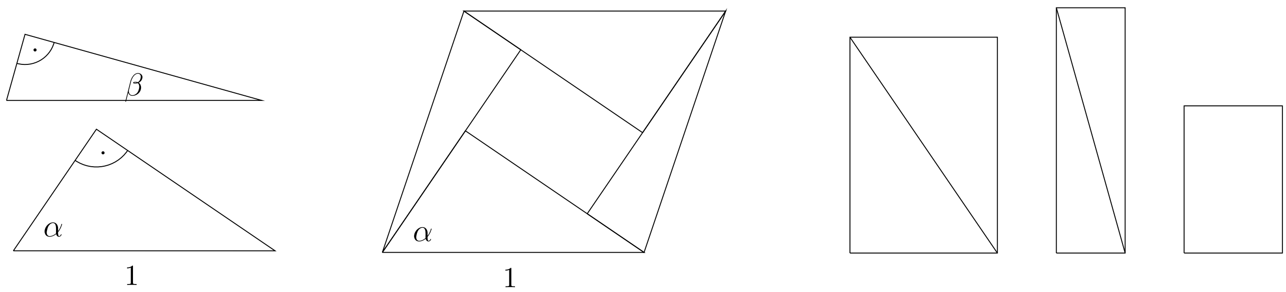
Additionstheoreme



1. $\sin 2\alpha \;=\; 2\sin\alpha\,\cos\alpha$ (gleichgroße Flächen, $h = \sin 2\alpha$ Höhe der Raute)



2. $\sin\,(\alpha+\beta) \;=\; \sin\alpha\,\cos\alpha + \sin\beta\,\cos\beta + (\sin\;\alpha - \sin\;\beta\,)\,(\cos\beta - \cos\alpha\,) \;=\; \sin\alpha\,\cos\beta + \sin\beta\,\cos\alpha$

weitere Formeln:

$$\begin{aligned}
\sin\,(\alpha-\beta) &= \sin\alpha\,\cos\beta \,-\, \sin\beta\,\cos\alpha\\
\cos\,(\alpha+\beta) &= \cos\alpha\,\cos\beta \,-\, \sin\alpha\,\sin\beta\\
\cos\,(\alpha-\beta) &= \cos\alpha\,\cos\beta \,+\, \sin\alpha\,\sin\beta\\
\sin\;\alpha \,+\, \sin\;\beta &= 2\;\sin\;\frac{\alpha+\beta}{2}\,\cos\;\frac{\alpha-\beta}{2}\\
\cos\;\alpha \,+\, \cos\;\beta &= 2\;\cos\;\frac{\alpha+\beta}{2}\,\cos\;\frac{\alpha-\beta}{2}
\end{aligned}$$

Ableitungen:

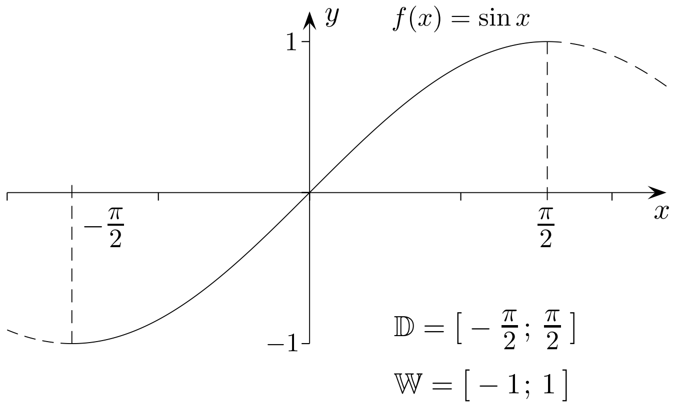
$$(\sin x)' \;=\; \lim_{h\to 0}\frac{\sin\,(x+h)-\sin x}{h} \;=\; \lim_{h\to 0}\frac{2\cos\,\dfrac{2x+h}{2}\;\sin\,\dfrac{h}{2}}{h} \;=\; \lim_{h\to 0}\,\cos\,(x+\frac{h}{2})\;\frac{\sin\;\dfrac{h}{2}}{\dfrac{h}{2}} \;=\; \cos x$$

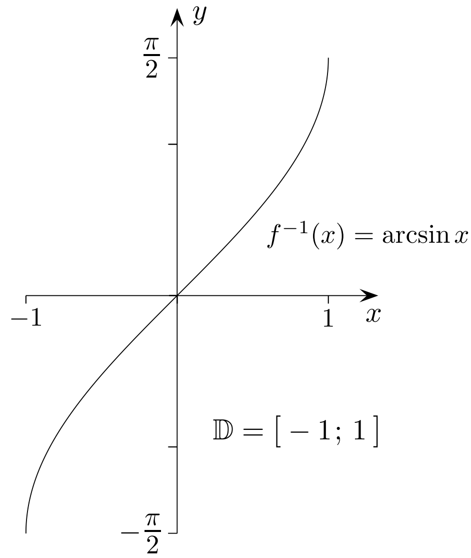
$$(\cos x)' \;=\; \lim_{h\to 0}\frac{\cos\,(x+h)-\cos x}{h} \;=\; \lim_{h\to 0}\frac{-2\sin\,\dfrac{2x+h}{2}\;\sin\,\dfrac{h}{2}}{h} \;=\; -\lim_{h\to 0}\,\sin\,(x+\frac{h}{2})\;\frac{\sin\;\dfrac{h}{2}}{\dfrac{h}{2}} \;=\; -\sin x$$

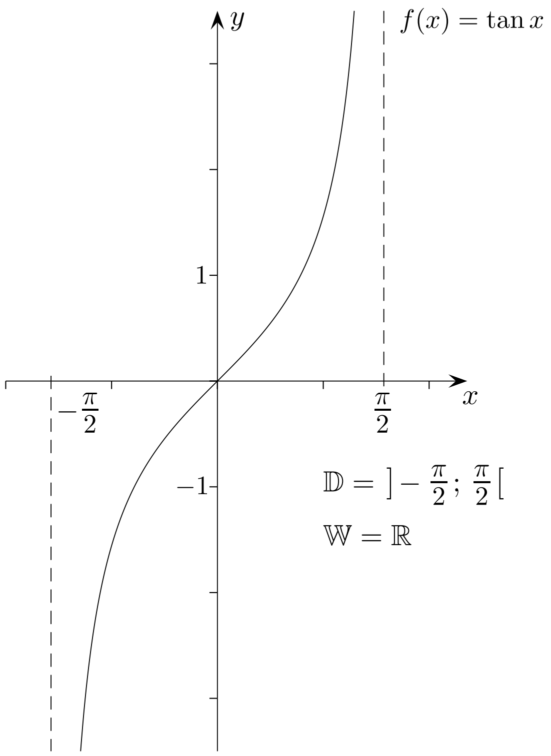
3. $\lim\limits_{x\to 0}\dfrac{\sin x}{x} \;=\; 1$ 4. $\tan x = \dfrac{\sin x}{\cos x}$ $(\tan x)' = \dfrac{1}{\cos^2 x}$

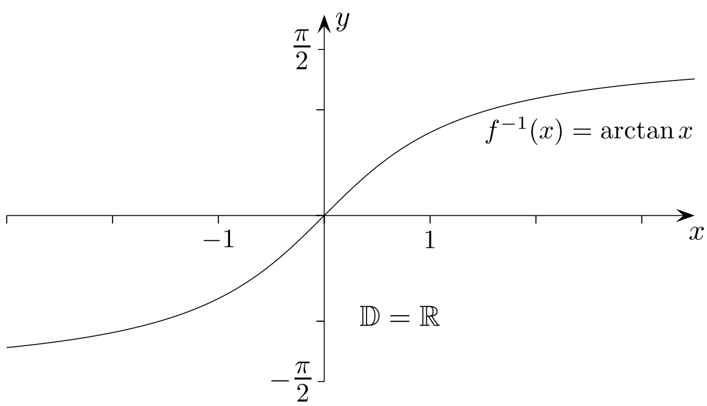
Arcus-Sinus und Arcus-Tangens

Der Definitionsbereich trigonometrischer Funktionen kann soweit eingeschränkt werden, dass die eingeschränkte Funktion streng monoton und damit umkehrbar ist.





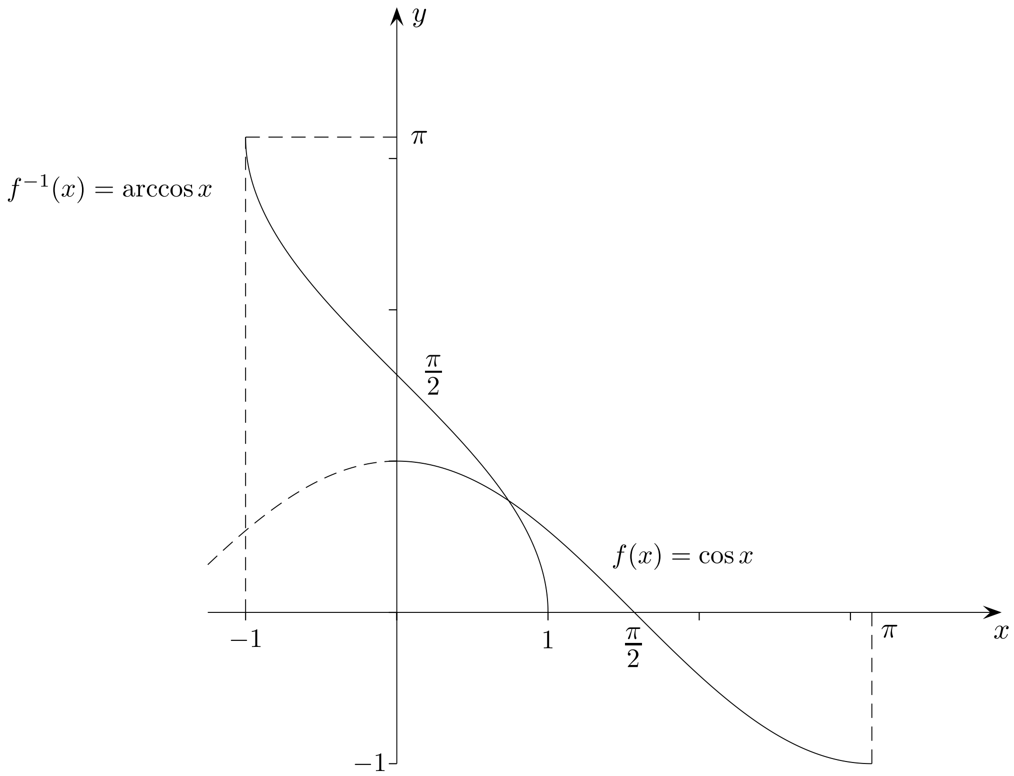




Zeige: $(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

Tipp: Wende auf $\sin(\arcsin(x)) = x$ die Kettenregel an und beachte, dass $\cos(x) = \sqrt{1-(\sin(x))^2}$ gilt.

Arcus-Cosinus



Ableitung $\arctan(x)$

Erläutere Folgendes:

$$\begin{aligned} && \tan(x) &= \tfrac{\sin(x)}{\cos(x)} \\ \Longrightarrow && \big(\tan(x)\big)' &= 1+\tan^2(x) \end{aligned}$$

$$\begin{aligned} && \tan(\arctan(x)) &= x \qquad\qquad | \ (\ )' \\ \Longrightarrow && \big[1+\underbrace{\tan^2(\arctan(x))}_{=x^2}\big]\cdot\big(\arctan(x)\big)' &= 1 \\ \Longrightarrow && \big(\arctan(x)\big)' &= \frac{1}{1+x^2} \end{aligned}$$